Relativistic and wide-angle corrections to galaxy power spectra

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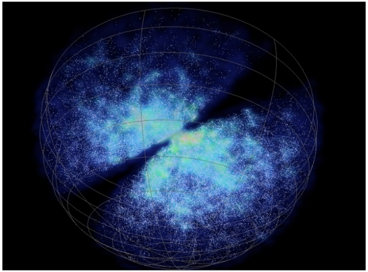
Introduction

- Galaxy power spectrum $P_g(z, k)$ studies use:
  - flat-sky approximation

- Next-generation surveys with wide sky area coverage, need:
  - wide-angle corrections
  - relativistic corrections

- Methodology for deriving wide angle corrections:
  - perturbative approach adopted by Noorikuhani & Scoccimarro, [arXiv:2207.12383]

Surveys



- Next-generation galaxy surveys
- Wide sky area

Overview

Fourier kernels

Redshift-space number density contrast:

$$\Delta(\boldsymbol{x}) = b\,\delta(\boldsymbol{x}) - \frac{1}{\mathcal{H}}\partial_\parallel^2 V(\boldsymbol{x}) + \Delta^{\rm corr}(\boldsymbol{x}) \ .$$

In Fourier space:

$$\Delta(\boldsymbol{k}) = \mathcal{K}(\boldsymbol{k})\,\delta(\boldsymbol{k}) \ .$$

In this work:

$$\mathcal{K} = \mathcal{K}^{\rm N} + \mathcal{K}^{\rm D} + \mathcal{K}^{\Phi} \ ,$$

where

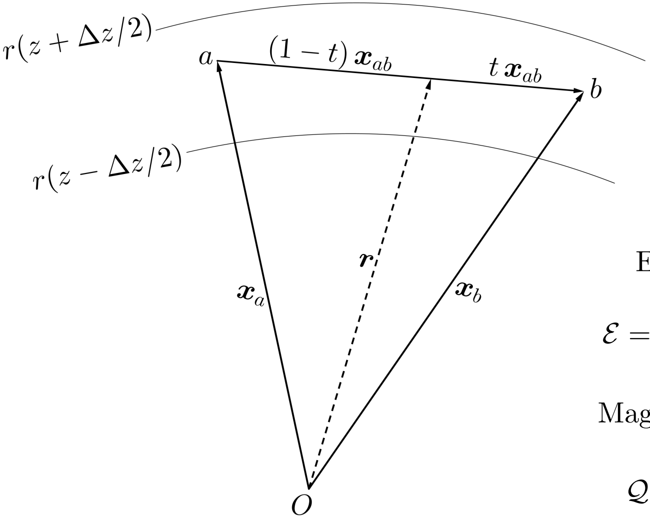
$$\mathcal{K}^{\rm N} = b + f\left(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{x}}\right)^2$$

$$\mathcal{K}^{\rm D} = {\rm i}\,\frac{\left(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{x}}\right)}{k}\mathcal{H}f\left[\mathcal{E} - 2\mathcal{Q} + \frac{2\left(\mathcal{Q}-1\right)}{\mathcal{H}}\frac{1}{x} - \frac{\mathcal{H}'}{\mathcal{H}^2}\right]$$

$$\mathcal{K}^{\Phi} = \frac{\mathcal{H}^2}{k^2}\left\{f(3-\mathcal{E}) + \frac{3}{2}\Omega_m\left[2 + \mathcal{E} - f - 4\mathcal{Q} + \frac{2\left(\mathcal{Q}-1\right)}{\mathcal{H}}\frac{1}{x} - \frac{\mathcal{H}'}{\mathcal{H}^2}\right]\right\}$$

Note that lensing will be included in future work.

Perturbative calculation



Geometry for a pair of fluctuations

Evolution bias:

$$\mathcal{E} = -\frac{\partial \ln n_g}{\partial \ln(1+z)}\,,$$

Magnification bias:

$$\mathcal{Q} = -\frac{\partial \ln n_g}{\partial \ln L_c}\,.$$

Comoving position vectors of observed galaxies:

$$\boldsymbol{x}_a = \boldsymbol{r} - (1-t)\,\boldsymbol{x}_{ab} \qquad \text{and} \qquad \boldsymbol{x}_b = \boldsymbol{r} + t\,\boldsymbol{x}_{ab}\ , \qquad 0 \le t \le 1\ .$$

$\boldsymbol{r}$: Choice of line-of-sight vector from O to the separation vector:

$$\boldsymbol{x}_{ab} = \boldsymbol{x}_a - \boldsymbol{x}_b.$$

Perturbation parameter vector:

$$\boldsymbol{\epsilon} = \frac{\boldsymbol{x}_{ab}}{r}$$

Plane-parallel limit: $\boldsymbol{\epsilon} \to \mathbf{0}$

Expansion in terms of $\boldsymbol{\epsilon}$ up to order 2: $\hat{\boldsymbol{x}}_a$, $\hat{\boldsymbol{x}}_b$, x_a^{-1}, x_b^{-1}

For example, $$\hat{\boldsymbol{x}}_b = \Big[1 - t\big(\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{r}}\big) + \tfrac{3}{2}t^2\big(\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{r}}\big)^2 - \tfrac{1}{2}t^2\epsilon^2\Big]\hat{\boldsymbol{r}} + t\big[1 - t\big(\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{r}}\big)\big]\boldsymbol{\epsilon} + O(\epsilon^3)\ .$$

The local Fourier power spectra are

$$P_{ab}(\boldsymbol{k},\boldsymbol{r}) = \int \frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3} \int \mathrm{d}^3\boldsymbol{\epsilon}\; r\, \mathrm{e}^{-\mathrm{i}\, r(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{\epsilon}}\, \mathcal{K}_a(\boldsymbol{k}',\boldsymbol{\epsilon},\boldsymbol{r})\, \mathcal{K}_b^*(\boldsymbol{k}',\boldsymbol{\epsilon},\boldsymbol{r})\, P(k')\ ,$$

where P is the linear matter power spectrum.

Local Fourier power spectra

$$P_{ab} = P_{ab}^{\mathrm{S}} + P_{ab}^{\mathrm{R}} + P_{ab}^{\mathrm{W}} + P_{ab}^{\mathrm{RW}}\ ,$$

where P_{ab}^{S} is the standard Kaiser power spectrum and the relativistic wide-angle corrections are:

$$P_{ab}^{\mathrm{I}}(\boldsymbol{k},\boldsymbol{r}) = P_{ab,0}^{\mathrm{I}} + \frac{1}{kr}P_{ab,1}^{\mathrm{I}} + \frac{1}{(kr)^2}P_{ab,2}^{\mathrm{I}} + O\left(1/(kr)^3\right)\ , \quad \mathrm{I} = \mathrm{W}, \mathrm{RW}$$

In general, P_{ab}^{I} are complex.

Multipole expansion and Surveys specifications

Multipole expansion

Decomposition of the power spectra into multipoles :

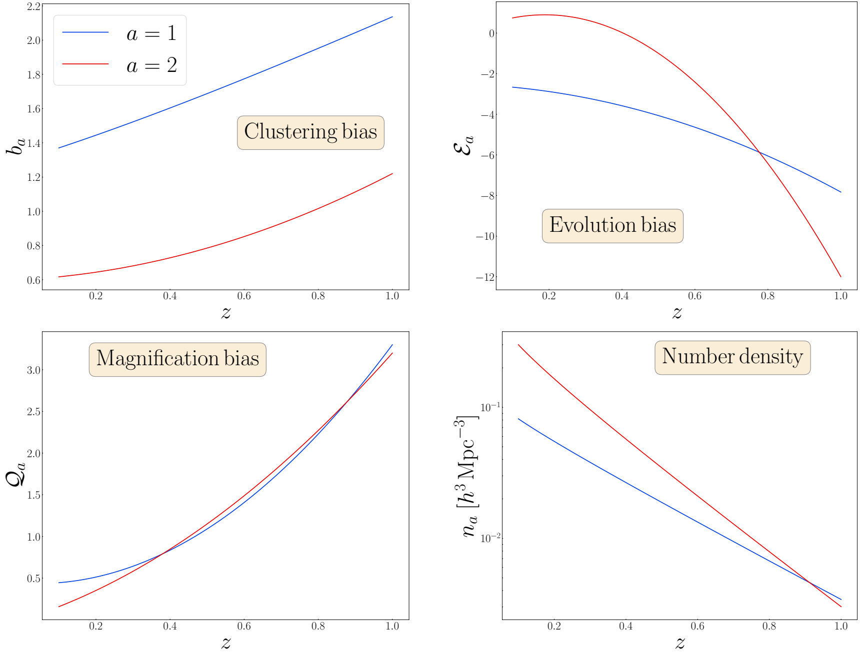
$$P^{\mathrm{I}}_{ab}(\boldsymbol{k},\boldsymbol{r}) = \sum_{\ell} P^{\mathrm{I}\,(\ell)}_{ab}(k,r)\,\mathcal{L}_\ell(\mu)\ ,$$

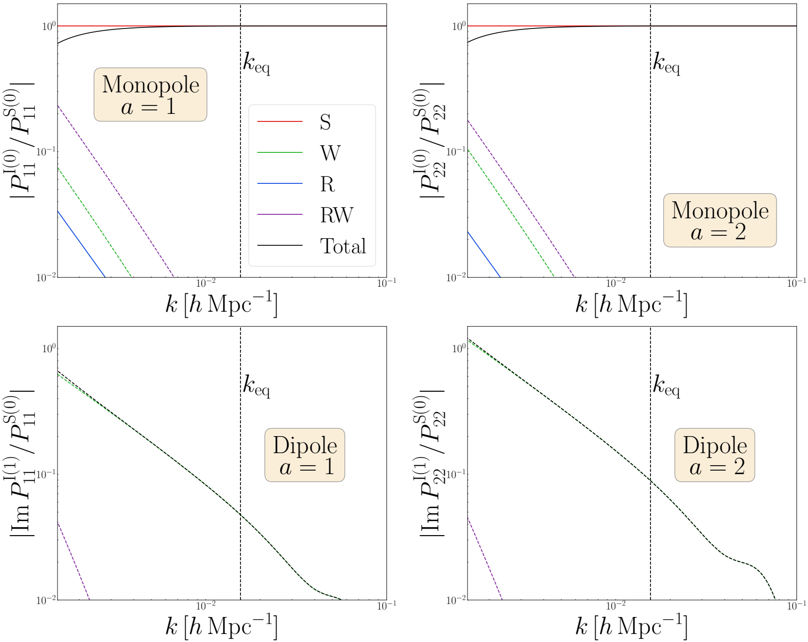
$\mathcal{L}_\ell$: Legendre polynomials, I = S, R, W, RW or R+W+RW,

Surveys specifications

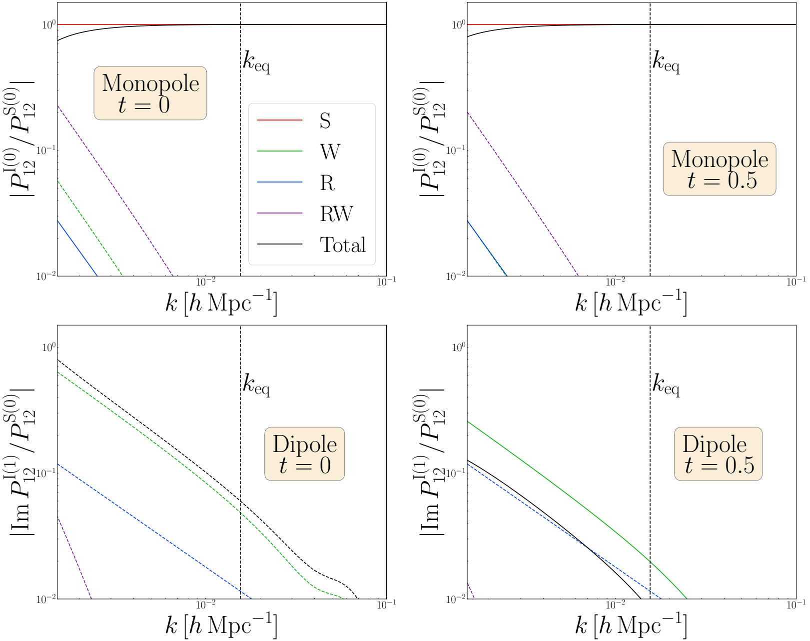
Two futuristic mock samples: $0.1 \le z \le 1$ and $\Omega_{\rm sky} = 15\,000\ \deg^2$

- $a = 1$: a futuristic 'beyond-DESI' BGS sample ($m_c = 22$).
- $a = 2$: a futuristic SKA Phase 2 HI galaxy sample ($S_c = 5\,\mu$Jy).


$a = 1$
$a = 2$
Clustering bias
b_a
z
Evolution bias
$\mathcal{E}_a$
Magnification bias
$\mathcal{Q}_a$
Number density
$n_a\ [h^3\,\mathrm{Mpc}^{-3}]$



Auto-power: Fractional correction at $z=0.5$, for $t=0$.



Cross-power: Fractional correction at $z = 0.5$, for $t = 0$ and $t = 0.5$.

Statistical significance of measuring effect I

The χ^2 for effect I in a redshift bin centred at z_i and the χ^2 cumulative over redshift bins:

$$\chi^2(z_i)^{\rm I} = \sum_{k,\mu} \frac{\left|P^{\rm I}_{ab}(z_i,k,\mu)\right|^2}{{\rm Var}\left[P_{ab}(z_i,k,\mu)\right]} \,, \quad \chi^2(\leq z_i)^{\rm I} = \sum_{j=1}^{i} \chi^2(z_j)^{\rm I} \,,$$

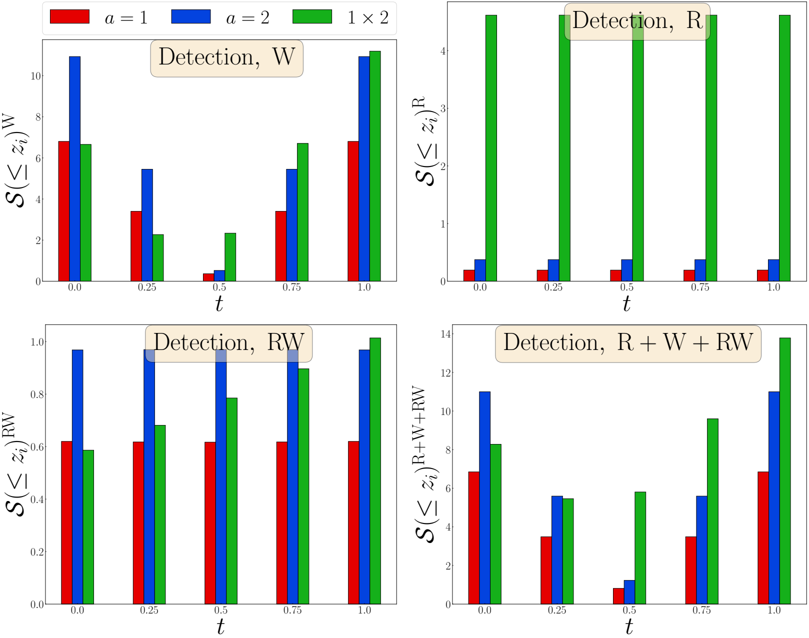
where ${\rm Var}\left[P_{ab}\right] = \frac{1}{N_{\boldsymbol{k}}} \left[\left|\tilde{P}_{ab}\right|^2 + \left|\tilde{P}_{aa}\right|\left|\tilde{P}_{bb}\right| \right]$,

$$\tilde{P}_{ab} = P_{ab} + \frac{\delta^{\rm K}_{ab}}{n_a} \,, \quad P_{ab} = P^{\rm S}_{ab} + P^{\rm R+W+RW}_{ab} \,, \quad N_{\boldsymbol{k}} = \frac{2\,\pi\,k^2\,\Delta k\,\Delta\mu}{k_{\rm f}^3} \,.$$

The cumulative detection significance for effect I:

$$\mathcal{S}(\leq z_i)^{\rm I} = \left[\chi^2(\leq z_i)^{\rm I}\right]^{1/2},$$

For numerical calculations: $\Delta z = 0.1$, $\Delta\mu = 0.04$, $\Delta k = k_{\rm f}$, $k_{\rm max} = 0.08\,(1+z)^{2/(2+n_s)}\ h\,{\rm Mpc}^{-1}$.



Cumulative detection significance $\mathcal{S}$ against t.

Bias on the best fit of $f_{\rm NL}$

Futuristic SKA2 HI galaxy survey: $0.1 \leq z \leq 2$, $\Omega_{\rm sky} = 30,000\,{\rm deg}^2$.

Total galaxy power spectrum with relativistic wide-angle corrections:

$$P_g = P_g^{\rm S} + \varepsilon P_g^{\rm corr}\,, \quad \text{where} \quad \varepsilon = 0, 1$$

Bias on the value of $f_{\rm NL}$, within a Gaussian Fisher formalism:

$$\delta f_{\rm NL} = f_{\rm NL}^{\rm true} - f_{\rm NL}^{\rm wrong} = -\left(\overset{0}{F}{}^{-1}\right)_{f_{\rm NL}\alpha} \overset{1}{F}_{\alpha\varepsilon}\,\delta\varepsilon\,.$$

For a simple optimistic estimate, we fix all cosmological and astrophysical parameters.

Bias on the estimate of $f_{\rm NL}$

$$\delta f_{\rm NL} = 0.66\,\sigma$$

where σ is the conditional error on $f_{\rm NL}$. **Note that this bias excludes the effect of lensing in the power spectrum.**

Conclusion

- Calculation of the relativistic wide-angle corrections to the Newtonian plane-parallel power spectra with arbitrary lines of sight for two tracers.

- Detection significance for the total relativistic wide-angle effects that ranges from $\sim 5\sigma$ up to 14σ, depending on the line-of-sight configuration.

- Detectability means that constraints on cosmological parameters could be affected by omitting these corrections to the standard power spectra (e.g., $f_{\rm NL}$).

- Future work will look at Fisher forecasts on $f_{\rm NL}$ and other cosmological parameters, and the bias on the best fit of $f_{\rm NL}$, including the lensing effect.

Thank you!