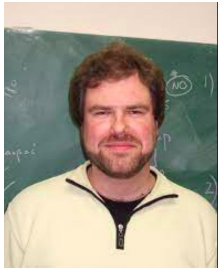
Relativistic matter bispectrum of cosmic structure



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Montandon et al (2404.02783)
Montandon et al (2212.06799)
Adamek et al (2110.11249)

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Relativistic matter bispectrum of cosmic structure

- Why?

Relativistic matter bispectrum of cosmic structure

- Why?
  - LCDM says it
  - it is annoying for inflation

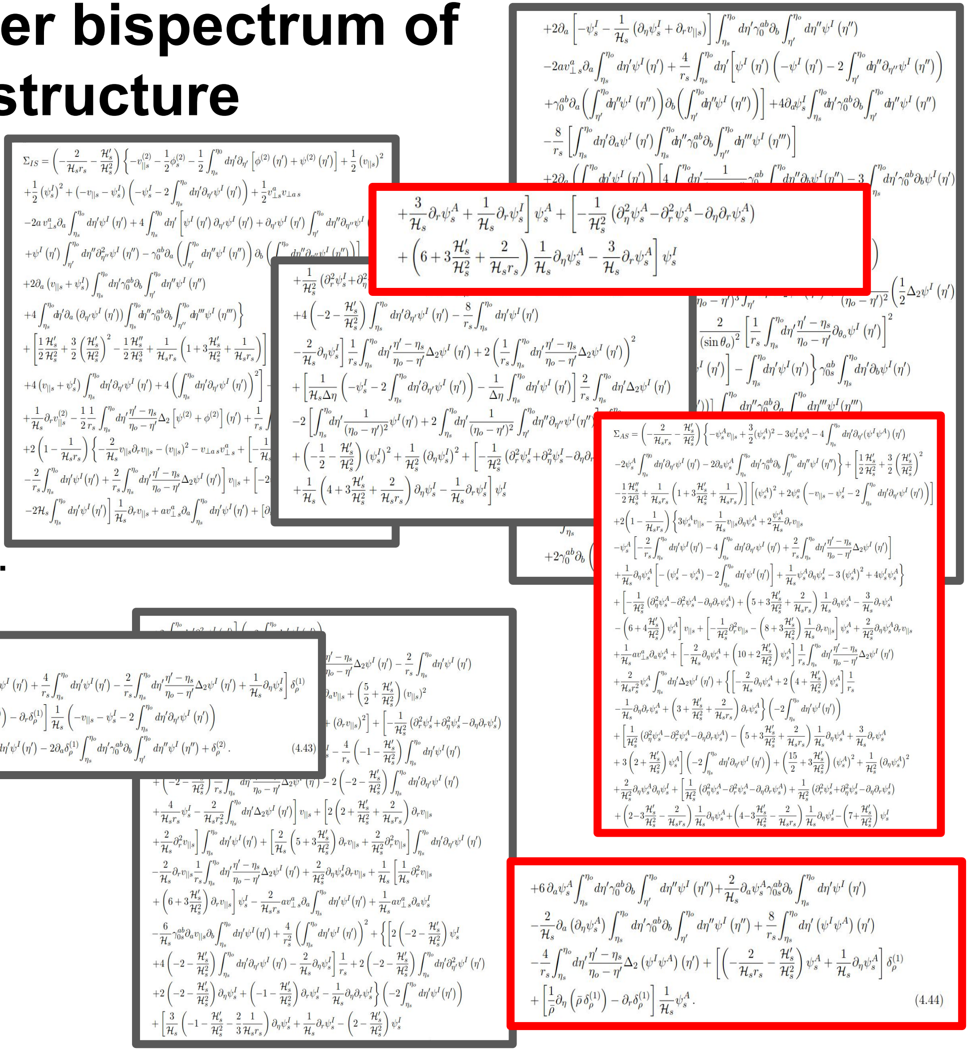
$$B(t,k_1,k_2,k_3) = T^{(1)}(k_1)T^{(1)}(k_2)T^{(1)}(k_3)B_{\mathcal{R}}(k_1,k_2,k_3) \\ +2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1,k_2,k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + \text{perm.}$$

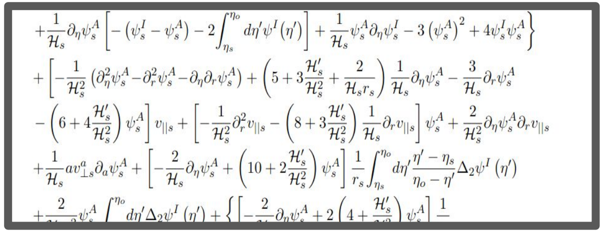
GR and Radiation effects are degenerate in time and in momentum space with PNG!

Relativistic matter bispectrum of cosmic structure

- Why?
  - LCDM says it
  - it is annoying for inflation

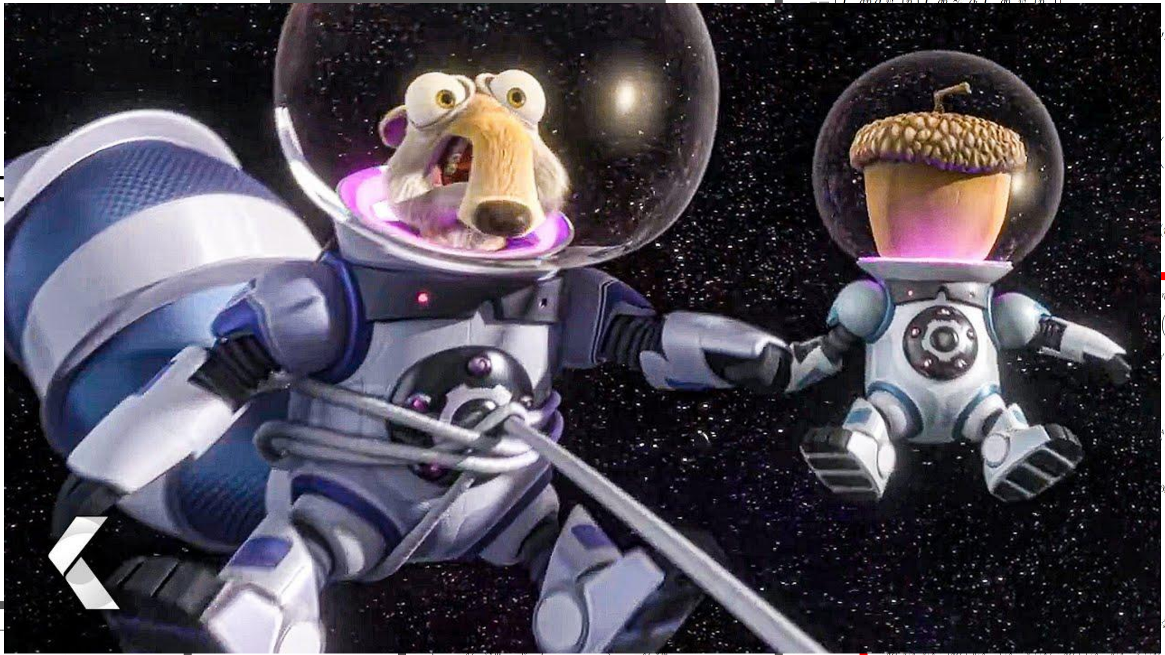
- How?
  - Computation is too hard…





Relativistic matter bispectrum of cosmic structure

- Why?
  - LCDM says it
  - it is annoying
- How?
  - Computation



$$+2\partial_a\left[-\psi^I_s-\frac{1}{\mathcal{H}_s}\left(\partial_\eta\psi^I_s+\partial_r v_{||s}\right)\right]\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)$$
$$-2av^a_{\perp s}\partial_a\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\frac{4}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\left[\psi^I\left(\eta'\right)\left(-\psi^I\left(\eta'\right)-2\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\right)\right.$$
$$\left.+\gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\right)\partial_b\left(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\right)\right]+4\partial_a\psi^I_s\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)$$

$$+\frac{1}{\mathcal{H}_s}\partial_\eta\psi^A_s\left[-\left(\psi^I_s-\psi^A_s\right)-2\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)\right]+\frac{1}{\mathcal{H}_s}\psi^A_s\partial_\eta\psi^I_s$$
$$+\left[-\frac{1}{\mathcal{H}_s^2}\left(\partial^2_\eta\psi^A_s-\partial^2_r\psi^A_s-\partial_\eta\partial_r\psi^A_s\right)+\left(5+3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}+\frac{2}{\mathcal{H}_s r_s}\right)\frac{1}{\mathcal{H}_s}\partial_\eta\psi^A_s-\frac{3}{\mathcal{H}_s}\partial_r\psi^A_s\right.$$
$$\left.-\left(6+4\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^A_s\right]v_{||s}+\left[-\frac{1}{\mathcal{H}_s^2}\partial^2_r v_{||s}-\left(8+3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\frac{1}{\mathcal{H}_s}\partial_r v_{||s}\right]\psi^A_s+\frac{2}{\mathcal{H}_s^2}\partial_\eta\psi^A_s\partial_r v_{||s}$$
$$+\frac{1}{\mathcal{H}_s}av^a_{\perp s}\partial_a\psi^A_s+\left[-\frac{2}{\mathcal{H}_s}\partial_\eta\psi^A_s+\left(10+2\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^A_s\right]\frac{1}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)$$

$$+\frac{4}{\mathcal{H}_s r_s}\psi^I_s-\frac{2}{\mathcal{H}_s r_s^2}\int_{\eta_s}^{\eta_o}d\eta'\Delta_2\psi^I\left(\eta'\right)\Big]v_{||s}+\left[2\left(2+\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}+\frac{2}{\mathcal{H}_s r_s}\right)\partial_r v_{||s}\right.$$
$$\left.+\frac{2}{\mathcal{H}_s}\partial^2_r v_{||s}\right]\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\left[\frac{2}{\mathcal{H}_s}\left(5+3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_r v_{||s}+\frac{2}{\mathcal{H}_s^2}\partial^2_r v_{||s}\right]\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)$$
$$-\frac{2}{\mathcal{H}_s}\partial_r v_{||s}\frac{1}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)+\frac{2}{\mathcal{H}_s^2}\partial_\eta\psi^I_s\partial_r v_{||s}+\frac{1}{\mathcal{H}_s}\left[\frac{1}{\mathcal{H}_s}\partial^2_r v_{||s}\right.$$
$$\left.+\left(6+3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_r v_{||s}\right]\psi^I_s-\frac{2}{\mathcal{H}_s r_s}av^a_{\perp s}\partial_a\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\frac{1}{\mathcal{H}_s}av^a_{\perp s}\partial_a\psi^I_s$$
$$-\frac{6}{\mathcal{H}_s}\gamma_{0s}^{ab}\partial_a v_{||s}\partial_b\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\frac{4}{r_s^2}\left(\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)\right)^2+\left\{\left[2\left(-2-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^I_s\right.\right.$$
$$\left.+4\left(-2-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)-\frac{2}{\mathcal{H}_s}\partial_\eta\psi^I_s\right]\frac{1}{r_s}+2\left(-2-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o}d\eta'\partial^2_{\eta'}\psi^I\left(\eta'\right)$$
$$\left.+2\left(-2-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_\eta\psi^I_s+\left(-1-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_r\psi^I_s-\frac{1}{\mathcal{H}_s}\partial_\eta\partial_r\psi^I_s\right\}\left(-2\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')\right)$$
$$+\left[\frac{3}{\mathcal{H}_s}\left(-1-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}-\frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\right)\partial_\eta\psi^I_s+\frac{1}{\mathcal{H}_s}\partial_r\psi^I_s-\left(2-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^I_s\right.$$

$$+\left(2-3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}-\frac{2}{\mathcal{H}_s r_s}\right)\frac{1}{\mathcal{H}_s}\partial_\eta\psi^A_s+\left(4-3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}-\frac{2}{\mathcal{H}_s r_s}\right)\frac{1}{\mathcal{H}_s}\partial_\eta\psi^I_s-\left(7+\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^I_s$$

$$+6\,\partial_a\psi^A_s\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)+\frac{2}{\mathcal{H}_s}\partial_a\psi^A_s\gamma_{0s}^{ab}\partial_b\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)$$
$$-\frac{2}{\mathcal{H}_s}\partial_a\left(\partial_\eta\psi^A_s\right)\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)+\frac{8}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\left(\psi^I\psi^A\right)\left(\eta'\right)$$
$$-\frac{4}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\left(\psi^I\psi^A\right)\left(\eta'\right)+\left[\left(-\frac{2}{\mathcal{H}_s r_s}-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^A_s+\frac{1}{\mathcal{H}_s}\partial_\eta\psi^A_s\right]\delta^{(1)}_\rho$$
$$+\left[\frac{1}{\bar\rho}\partial_\eta\left(\bar\rho\,\delta^{(1)}_\rho\right)-\partial_r\delta^{(1)}_\rho\right]\frac{1}{\mathcal{H}_s}\psi^A_s. \qquad (4.44)$$

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- Why?
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- How?
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  - SIMULATION!

Simulation pipeline

Inflation

Simulation pipeline

Inflation

Inflaton ϕ

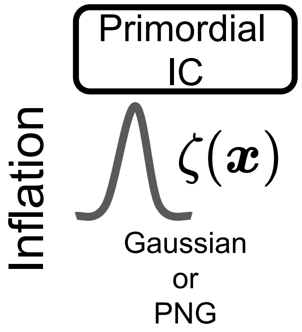
Simulation pipeline

Inflation

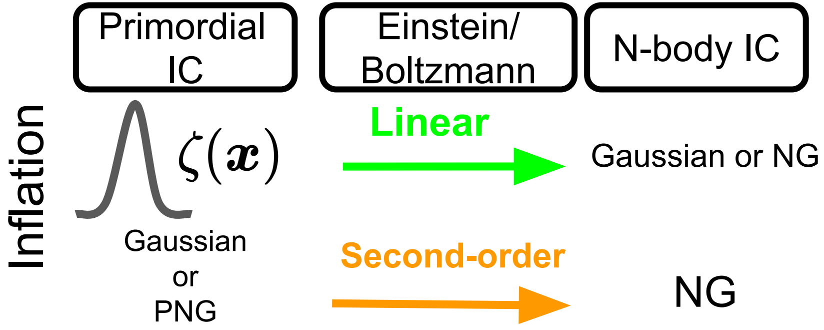
Inflaton ϕ

some other fancy field

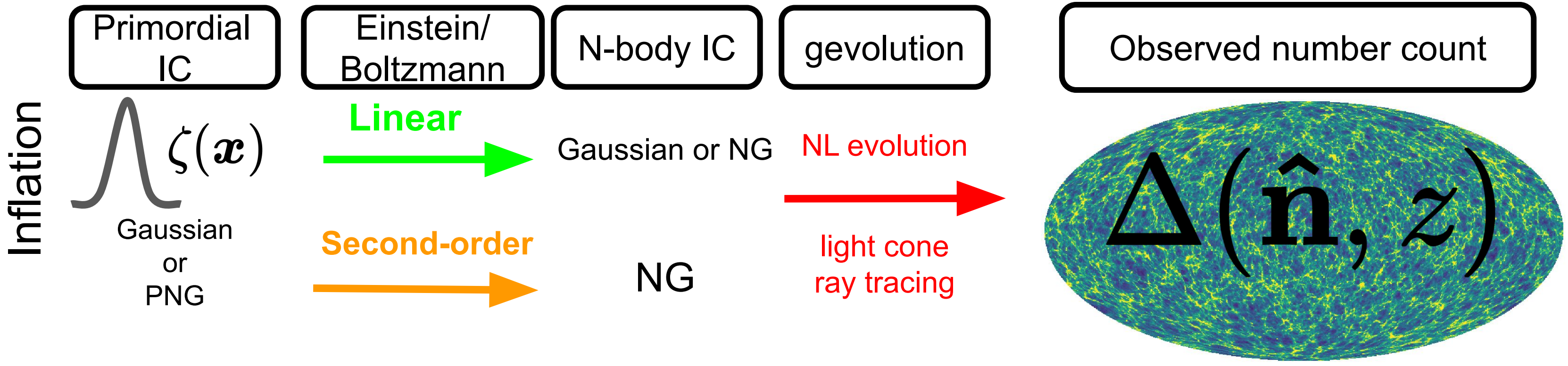
Simulation pipeline



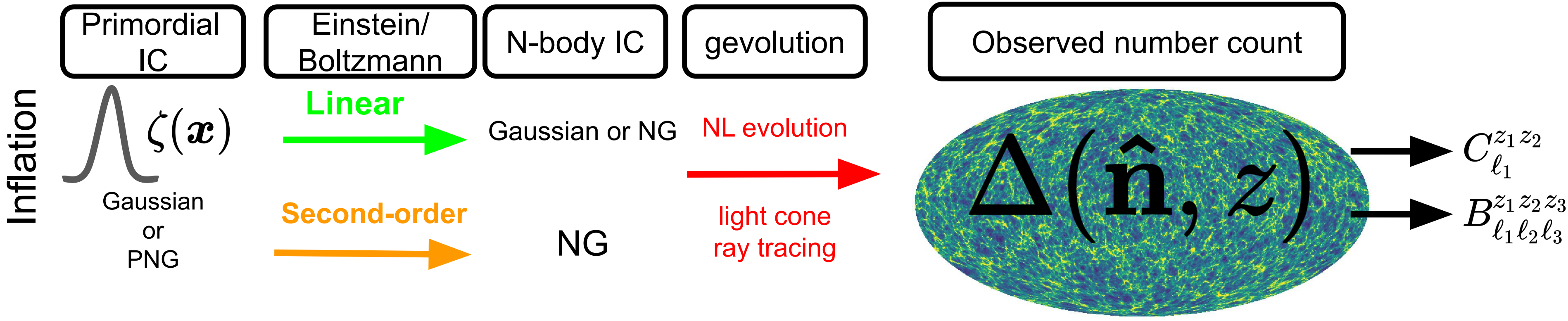
Simulation pipeline



Simulation pipeline

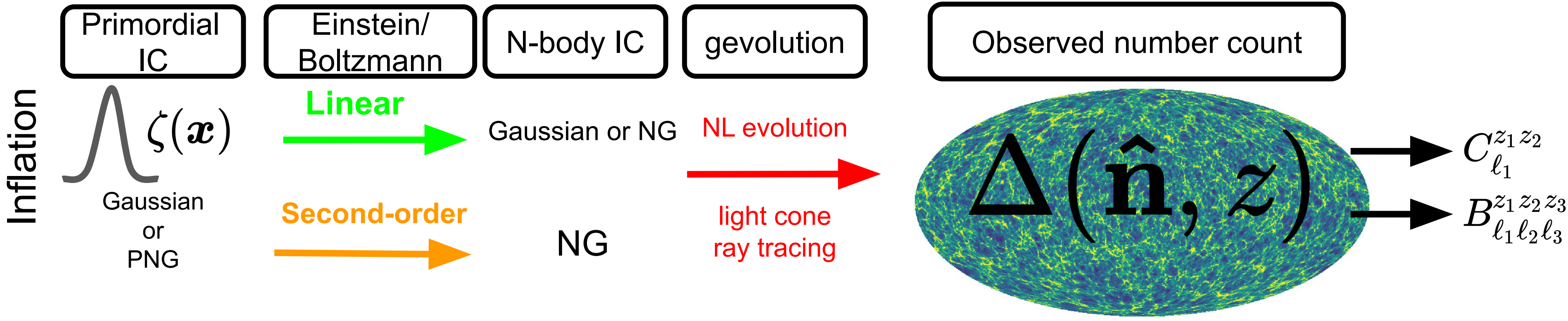


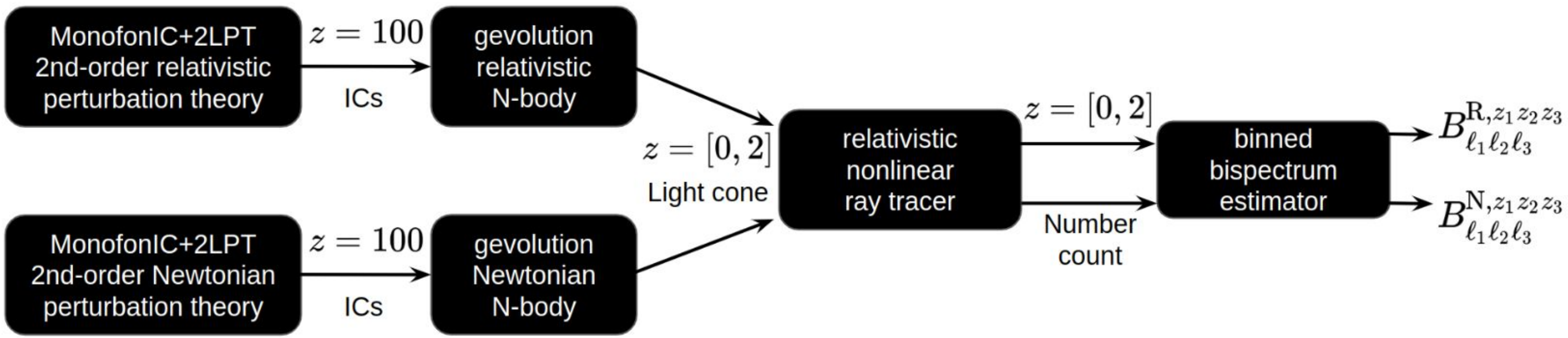
Simulation pipeline



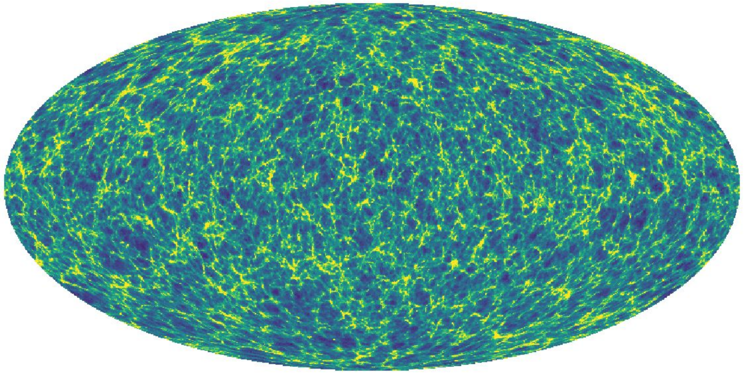
IMPORTANT: Gauge invariance!

Simulation pipeline

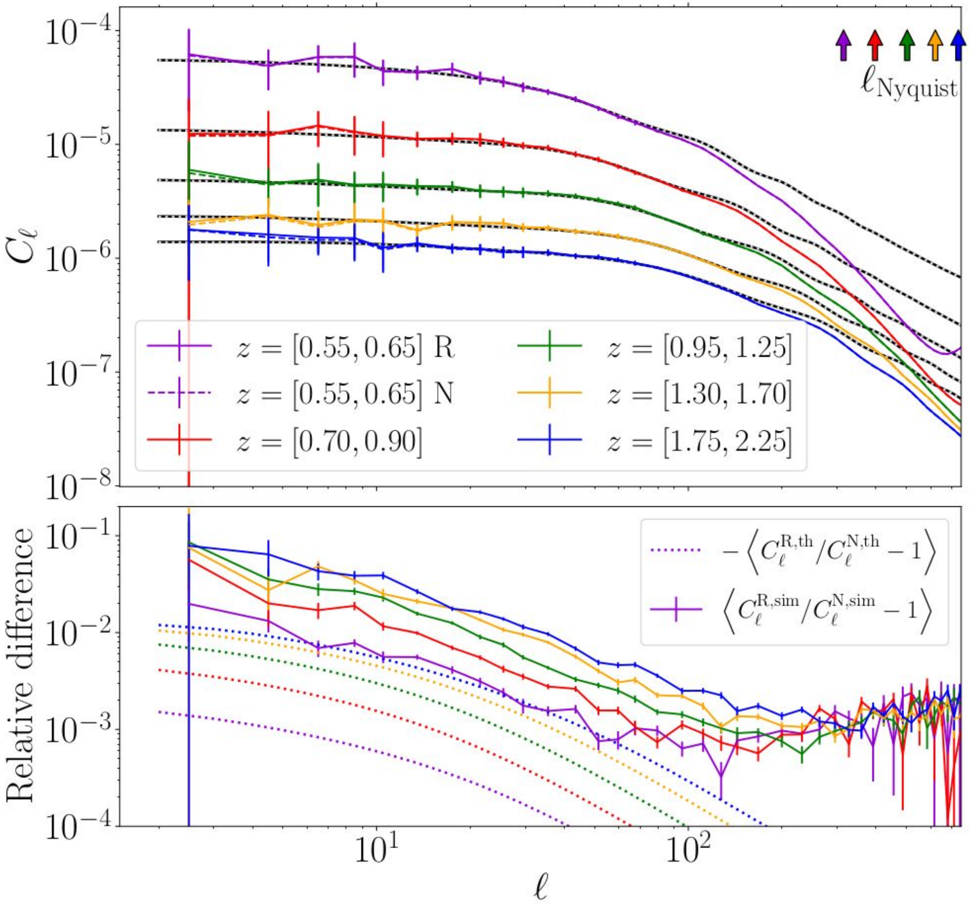




Results

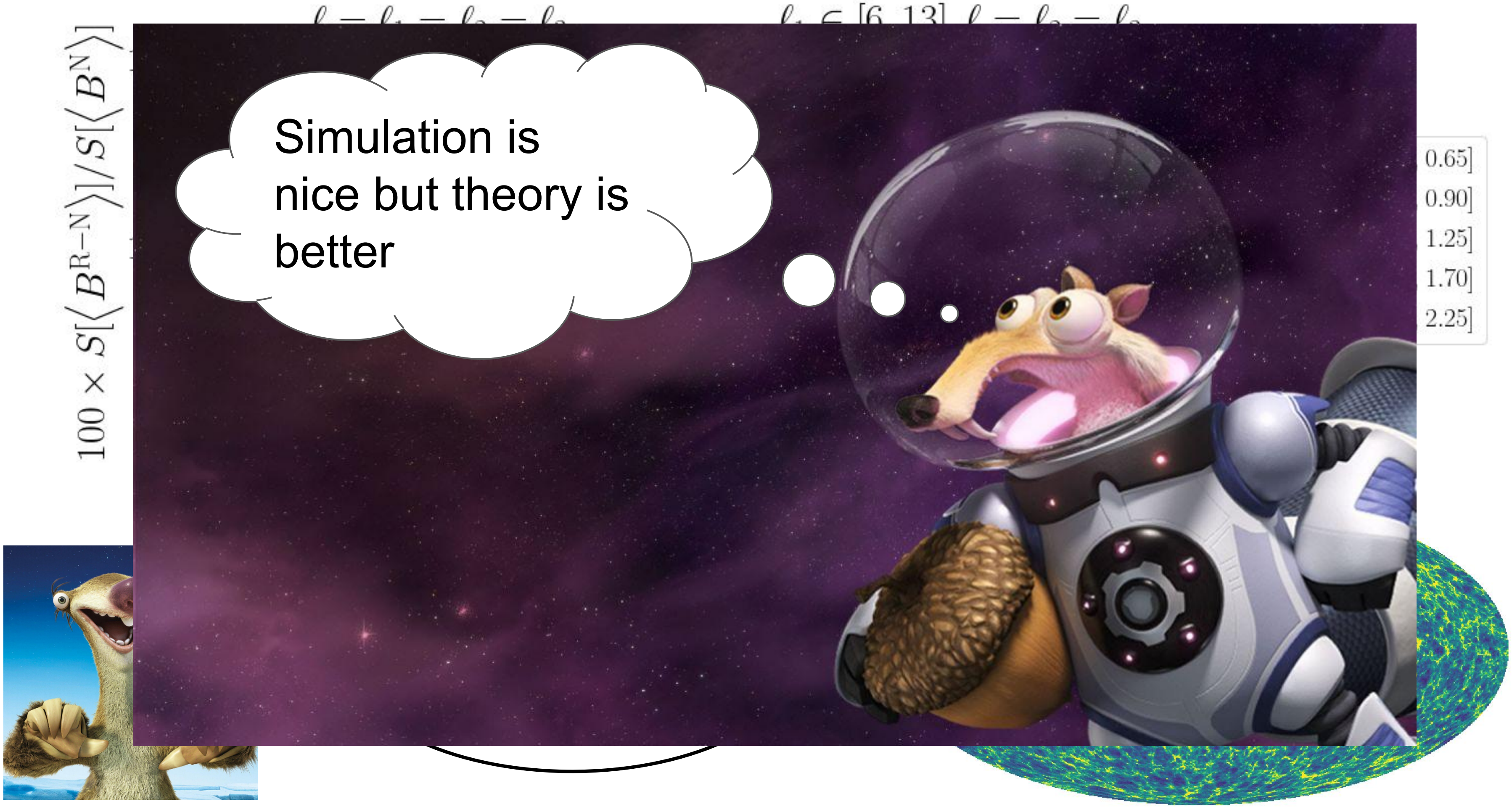


Nonlinear GR/Rad effects affect the power spectrum!
z=2: 1-10% effect

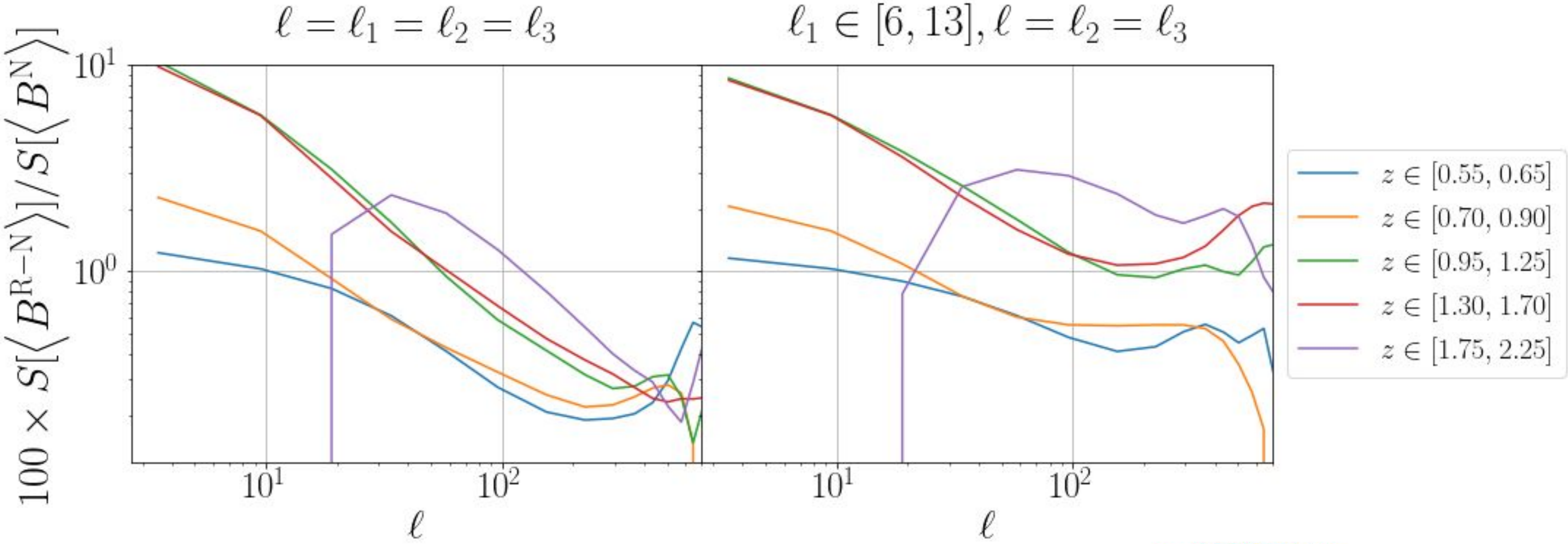


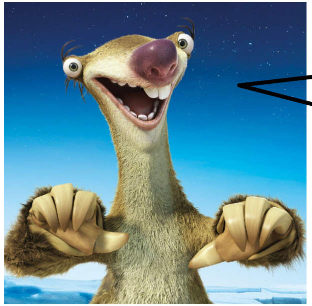
C_ℓ
$\ell_{\rm Nyquist}$
$z = [0.55, 0.65]$ R
$z = [0.55, 0.65]$ N
$z = [0.70, 0.90]$
$z = [0.95, 1.25]$
$z = [1.30, 1.70]$
$z = [1.75, 2.25]$
Relative difference
$-\left\langle C_\ell^{\rm R,th}/C_\ell^{\rm N,th} - 1\right\rangle$
$\left\langle C_\ell^{\rm R,sim}/C_\ell^{\rm N,sim} - 1\right\rangle$
ℓ

Results



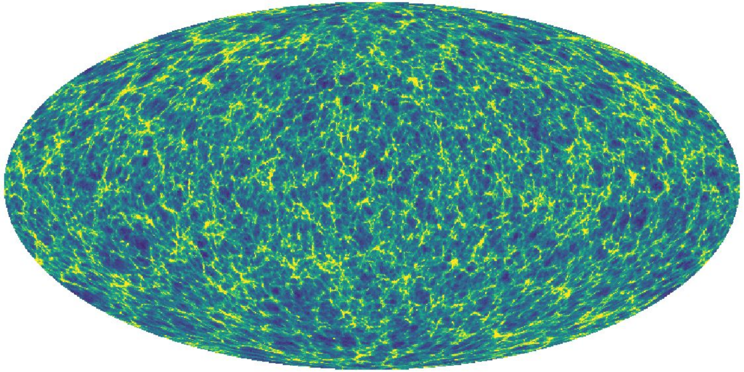
Results





GR/Rad effects affect the bispectrum!

z=2: 1-10% effect



Angular bispectrum calculation

- Redshift bins
- GR up to $\mathcal{H}^4/k^4$
- Radiation up to $\mathcal{H}^4/k^4$
- Projection effects up to $\mathcal{H}/k$

$$b_{\ell_1\ell_2\ell_3}^{r_1r_2r_3} = \frac{8}{\pi^3}\int dr'_1 dr'_2 dr'_3 W(r_1, r'_1)W(r_2, r'_2)W(r_3, r'_3)$$
$$\int dk_1 dk_2 dk_3 d\chi (k_1 k_2 k_3 \chi)^2 B^{r'_1 r'_2 r'_3}(k_1, k_2, k_3)$$
$$j_{\ell_1}(k_1 r'_1) j_{\ell_2}(k_2 r'_2) j_{\ell_3}(k_3 r'_3) j_{\ell_1}(k_1\chi) j_{\ell_2}(k_2\chi) j_{\ell_3}(k_3\chi)$$

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$$j_{\ell_1}(k_1 r'_1) j_{\ell_2}(k_2 r'_2) j_{\ell_3}(k_3 r'_3) j_{\ell_1}(k_1 \chi) j_{\ell_2}(k_2 \chi) j_{\ell_3}(k_3 \chi)$$

$$\begin{aligned} b^{\hat{z}_1 \hat{z}_2 \hat{z}_3}_{\ell_1 \ell_2 \ell_3} = {} & b^{\delta_2}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r^2 v_2}_{\ell_1 \ell_2 \ell_3} + b^{(\partial_r^2 v_1)^2}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \partial_r^3 v_1}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \partial_r \delta_1}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r^2 v_1 \delta_1}_{\ell_1 \ell_2 \ell_3} \\ & + b^{\partial_r v_2}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \partial_r^2 v_1}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \delta_2}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \dot{\delta}}_{\ell_1 \ell_2 \ell_3} + b^{v^a \partial_a \partial_r v_1}_{\ell_1 \ell_2 \ell_3} + b^{\psi_1 \partial_r^3 v_1}_{\ell_1 \ell_2 \ell_3} + b^{\psi_1 \partial_r \delta_1}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \partial_r^2 \psi_1}_{\ell_1 \ell_2 \ell_3} , \end{aligned} \tag{3.5}$$

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$$\int dk_1 dk_2 dk_3 d\chi (k_1 k_2 k_3 \chi)^2 B^{r'_1 r'_2 r'_3}(k_1, k_2, k_3)$$
$$j_{\ell_1}(k_1 r'_1) j_{\ell_2}(k_2 r'_2) j_{\ell_3}(k_3 r'_3) j_{\ell_1}(k_1 \chi) j_{\ell_2}(k_2 \chi) j_{\ell_3}(k_3 \chi)$$

$$\begin{aligned} b^{\hat{z}_1 \hat{z}_2 \hat{z}_3}_{\ell_1 \ell_2 \ell_3} = {} & b^{\delta_2}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r^2 v_2}_{\ell_1 \ell_2 \ell_3} + b^{(\partial_r^2 v_1)^2}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \partial_r^3 v_1}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \partial_r \delta_1}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r^2 v_1 \delta_1}_{\ell_1 \ell_2 \ell_3} \\ & + b^{\partial_r v_2}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \partial_r^2 v_1}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \delta_2}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \dot{\delta}}_{\ell_1 \ell_2 \ell_3} + b^{v^a \partial_a \partial_r v_1}_{\ell_1 \ell_2 \ell_3} + b^{\psi_1 \partial_r^3 v_1}_{\ell_1 \ell_2 \ell_3} + b^{\psi_1 \partial_r \delta_1}_{\ell_1 \ell_2 \ell_3} + b^{\partial_r v_1 \partial_r^2 \psi_1}_{\ell_1 \ell_2 \ell_3} , \end{aligned} \quad (3.5)$$

$$\begin{aligned} b^{\delta_2}_{\ell_1 \ell_2 \ell_3} = {} & 2 \sum_{mn} \int d\chi C^{(n,0,0)}_{\ell_2}(\chi) C^{(m,0,0)}_{\ell_3}(\chi) \\ & \left(\chi^2 \int \frac{dr_1}{r_1^2} D^2(r_1) W(r_1) \left[f^{(-4)}_{nm} I_{\ell_1}(-1, r_1, \chi) + f^{(-2)}_{nm} A_{\ell_1}(r_1, \chi) \right. \right. \\ & \left. + \sum_p c_p \left(f^{(-4,R)}_{nm} I_{\ell_1}(b + ip\eta - 1, r_1, \chi) + f^{(-2,R)}_{nm} I_{\ell_1}(b + ip\eta + 1, r_1, \chi) \right) \right] \\ & \left. + f^{(0)}_{nm} D^2(\chi) W(\chi) + \mathcal{D}_{\ell_1} \left[f^{(2)}_{nm} D^2(\chi) W(\chi) \right] + \mathcal{D}^2_{\ell_1} \left[f^{(4)}_{nm} D^2(\chi) W(\chi) \right] \right) . \end{aligned} \quad (3.14)$$

Angular bispectrum calculation

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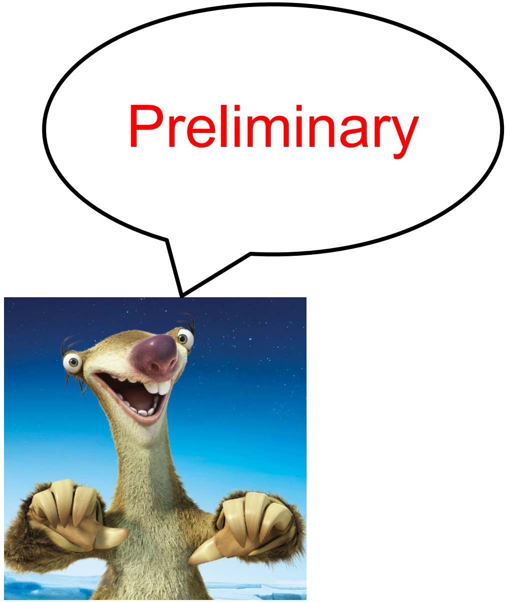

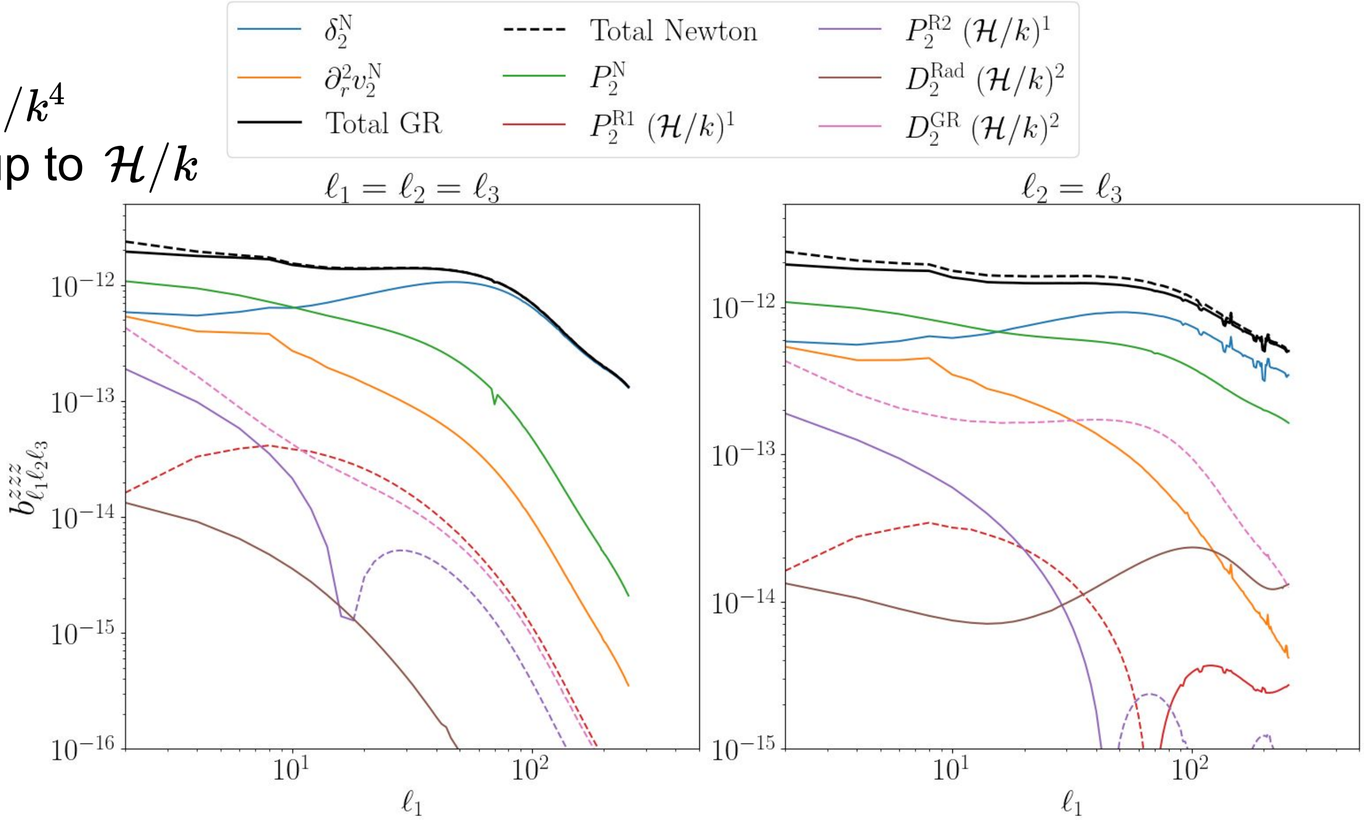

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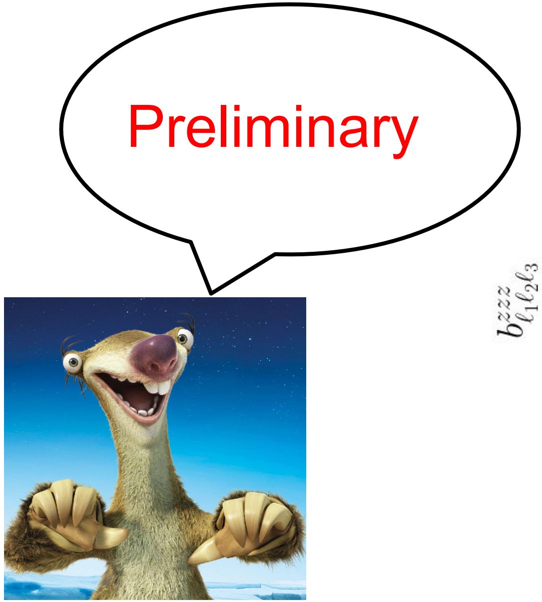

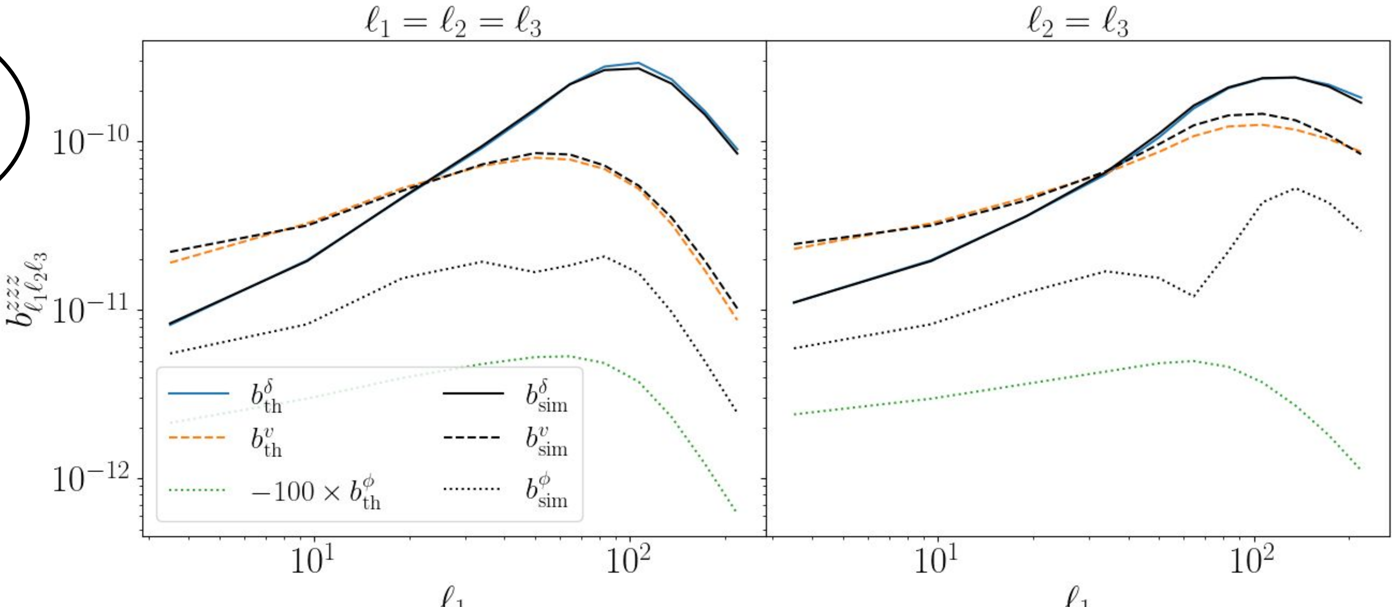

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$$b^{\rm GR}_{\rm sim} \stackrel{?}{=} b^{\rm GR}_{\rm th} \quad \text{and} \quad b^{\rm Rad}_{\rm sim} \stackrel{?}{=} b^{\rm Rad}_{\rm th}$$

Angular bispectrum calculation

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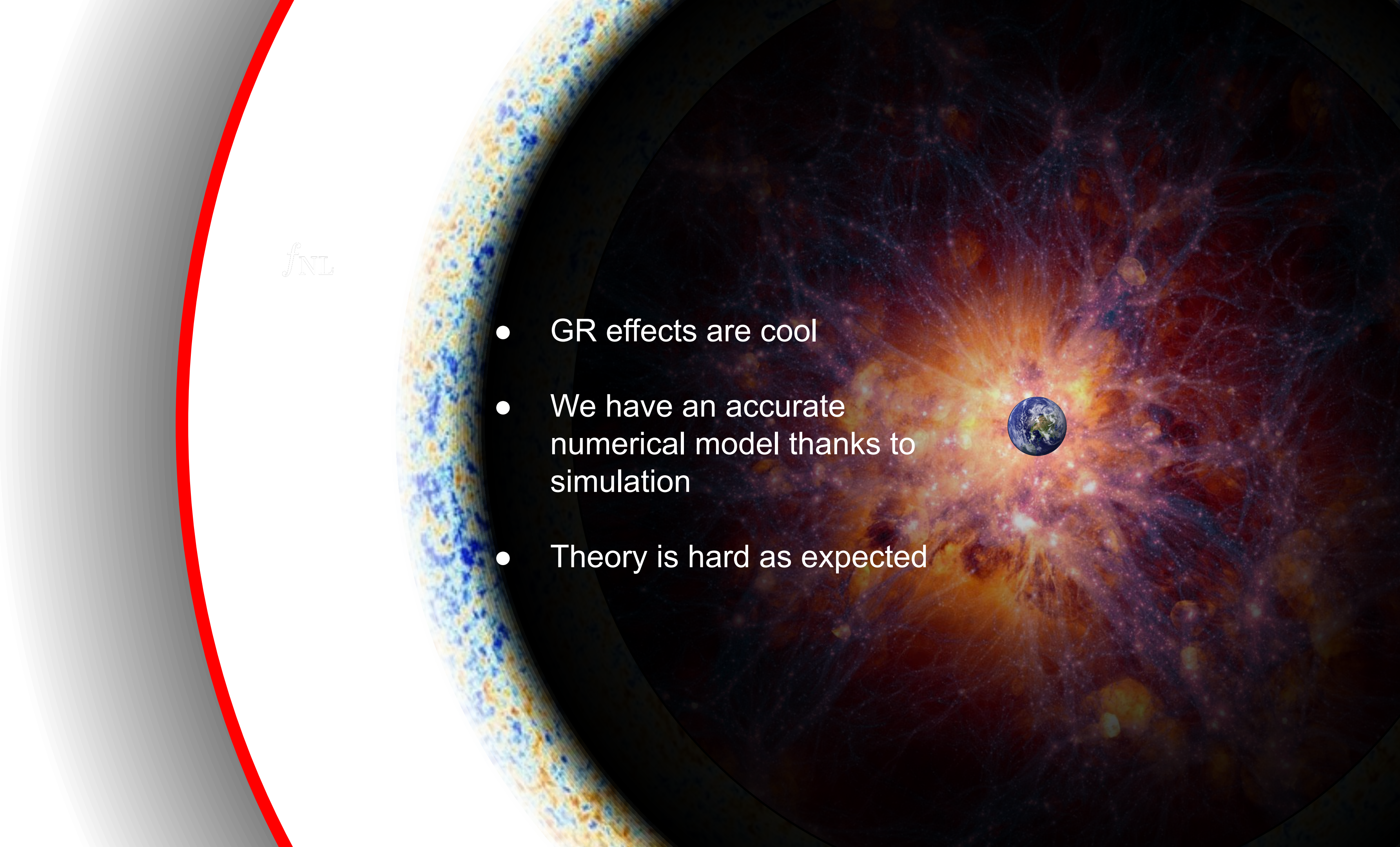
- GR up to $\mathcal{H}^4/k^4$
- Radiation up to $\mathcal{H}^4/k^4$
- Projection effects up to $\mathcal{H}/k$

$$b^{\rm GR}_{\rm sim} \stackrel{?}{=} b^{\rm GR}_{\rm th} \quad \text{and} \quad b^{\rm Rad}_{\rm sim} \stackrel{?}{=} b^{\rm Rad}_{\rm th}$$

Come back soon for the results, we are on it



- GR effects are cool
- We have an accurate numerical model thanks to simulation
- Theory is hard as expected


f_{NL}
Thank you !