

# Relativistic matter bispectrum of cosmic structure



Bartjan van Tent



Cornelius Rampf



Jorge Noreña



Enea Di Dio

Thomas Montandon

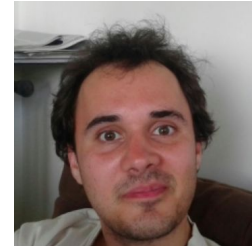
Montandon et al (2404.02783)

Montandon et al (2212.06799)

Adamek et al (2110.11249)



Julian Adamek



Clément Stahl



Oliver Hahn



UNIVERSITÉ DE  
MONTPELLIER



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# Relativistic matter bispectrum of cosmic structure

- Why?

# Relativistic matter bispectrum of cosmic structure

- Why?
  - LCDM says it
  - it is annoying for inflation

$$B(t, k_1, k_2, k_3) = T^{(1)}(k_1)T^{(1)}(k_2)T^{(1)}(k_3)B_{\mathcal{R}}(k_1, k_2, k_3) \\ + 2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1, k_2, k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + \text{perm.}$$

GR and Radiation effects are degenerate in time and in momentum space with PNG!

# Relativistic matter bispectrum of cosmic structure

- Why?
  - LCDM says it
  - it is annoying for inflation
- How?
  - Computation is too hard...

$$\begin{aligned}
 & + \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^A \left[ -(\psi_s^I - \psi_s^A) - 2 \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') \right] + \frac{1}{\mathcal{H}_s} \psi_s^A \partial_\eta \psi_s^I - 3(\psi_s^A)^2 + 4\psi_s^I \psi_s^A \Big\} \\
 & + \left[ -\frac{1}{\mathcal{H}_s^2} (\partial_\eta^2 \psi_s^A - \partial_\eta^2 \psi_s^A - \partial_\eta \partial_\eta \psi_s^A) + \left( 5 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^A - \frac{3}{\mathcal{H}_s} \partial_\eta \psi_s^A \right. \\
 & \left. - \left( 6 + 4 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^A \right] v_{||s} + \left[ -\frac{1}{\mathcal{H}_s^2} \partial_\eta^2 v_{||s} - \left( 8 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \frac{1}{\mathcal{H}_s} \partial_\eta v_{||s} \right] \psi_s^A + \frac{2}{\mathcal{H}_s^2} \partial_\eta \psi_s^A \partial_\eta v_{||s} \\
 & + \frac{1}{\mathcal{H}_s} \alpha v_{||s} \partial_\eta \psi_s^A + \left[ -\frac{2}{\mathcal{H}_s} \partial_\eta \psi_s^A + \left( 10 + 2 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^A \right] \frac{1}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 \psi^I(\eta') \\
 & + \frac{2}{\mathcal{H}_s} \psi_s^A \int_{\eta_s}^{\eta_0} d\eta' \Delta_2 \psi^I(\eta') + \left\{ \left[ -\frac{2}{\mathcal{H}_s} \partial_\eta \psi_s^A + 2 \left( 4 + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^A \right] \frac{1}{\mathcal{H}_s} \right.
 \end{aligned}$$

$$\begin{aligned}
 & -6 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') + \frac{4}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') - \frac{2}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 \psi^I(\eta') + \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^I \delta^{(1)} \\
 & + \left[ \frac{1}{2} \partial_\eta (\delta \delta^{(1)}) - \partial_\eta \delta^{(1)} \right] \frac{1}{\mathcal{H}_s} \left[ -v_{||s} - v_s^I - 2 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') \right] \\
 & + 2 \partial_\eta \delta^{(1)} \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') - 2 \partial_\eta \delta^{(1)} \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \psi^I(\eta') + \delta^{(2)}. \quad (4.3)
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{IS} = & \left( -\frac{2}{\mathcal{H}_s r_s} - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \left\{ -v_{||s}^I - \frac{1}{2} v_s^{(2)I} - \frac{1}{2} \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta [\psi^{(2)I}(\eta') + \psi^{(2)I}(\eta')] + \frac{1}{2} (v_{||s}^I)^2 \right. \\
 & + \frac{1}{2} (v_s^I)^2 + (-v_{||s} - v_s^I) \left. \left[ -v_s^I - 2 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') + \frac{1}{2} v_s^{I, \text{Lensing}} \right] \right. \\
 & - 2 v_s^I \partial_\eta \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') + 4 \int_{\eta_s}^{\eta_0} d\eta' [\psi^I(\eta') \partial_\eta \psi^I(\eta') + \partial_\eta \psi^I(\eta') \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta')] \\
 & + \psi^I(\eta') \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta^2 \psi^I(\eta') - \partial_\eta^2 \psi^I(\eta') \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') \Big\} \delta^{(1)} \left( \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \psi^I(\eta') \right) \\
 & + 2 \partial_\eta (v_{||s} + v_s^I) \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') \\
 & + 4 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta (\partial_\eta \psi^I(\eta')) \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \int_{\eta_s}^{\eta_0} d\eta'' \psi^I(\eta'') \\
 & + \left[ \frac{1}{2} \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + 3 \left( \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right)^2 - \frac{1}{2} \frac{\mathcal{H}''_s}{\mathcal{H}_s^3} - \frac{1}{\mathcal{H}_s r_s} \left( 1 + \frac{\mathcal{H}'_s}{\mathcal{H}_s} + \frac{1}{\mathcal{H}_s r_s} \right) \right] \\
 & + 4 (v_{||s} + v_s^I) \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') + 4 \left( \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') \right)^2 \\
 & + \frac{1}{\mathcal{H}_s} \partial_\eta (v_{||s}^I) \left[ \frac{1}{2} r_s \int_{\eta_s}^{\eta_0} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 [\psi^{(2)I} + \psi^{(2)I}] (\eta') + \frac{1}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \psi^{(2)I}(\eta') \right. \\
 & + 2 \left( 1 - \frac{1}{\mathcal{H}_s r_s} \right) \left\{ \frac{2}{\mathcal{H}_s} \partial_\eta v_{||s} v_{||s} - (v_{||s}^I)^2 - v_{||s} v_s^{I, \text{Lensing}} + \left[ \frac{1}{\mathcal{H}_s} \right. \right. \\
 & \left. \left. \frac{2}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') + \frac{2}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 \psi^I(\eta') \right] v_{||s} + \left[ -2 \right. \right. \\
 & \left. \left. - 2 \mathcal{H}_s \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') \right] \frac{1}{\mathcal{H}_s} \partial_\eta v_{||s} + \alpha v_{||s} \partial_\eta \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') + \beta \right.
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{3}{\mathcal{H}_s} \partial_\eta \psi_s^A + \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^I \Big\} \psi_s^A + \left[ -\frac{1}{\mathcal{H}_s^2} (\partial_\eta^2 \psi_s^A - \partial_\eta^2 \psi_s^A - \partial_\eta \partial_\eta \psi_s^A) \right. \\
 & \left. + \left( 6 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^A - \frac{3}{\mathcal{H}_s} \partial_\eta \psi_s^A \right] \psi_s^I
 \end{aligned}$$

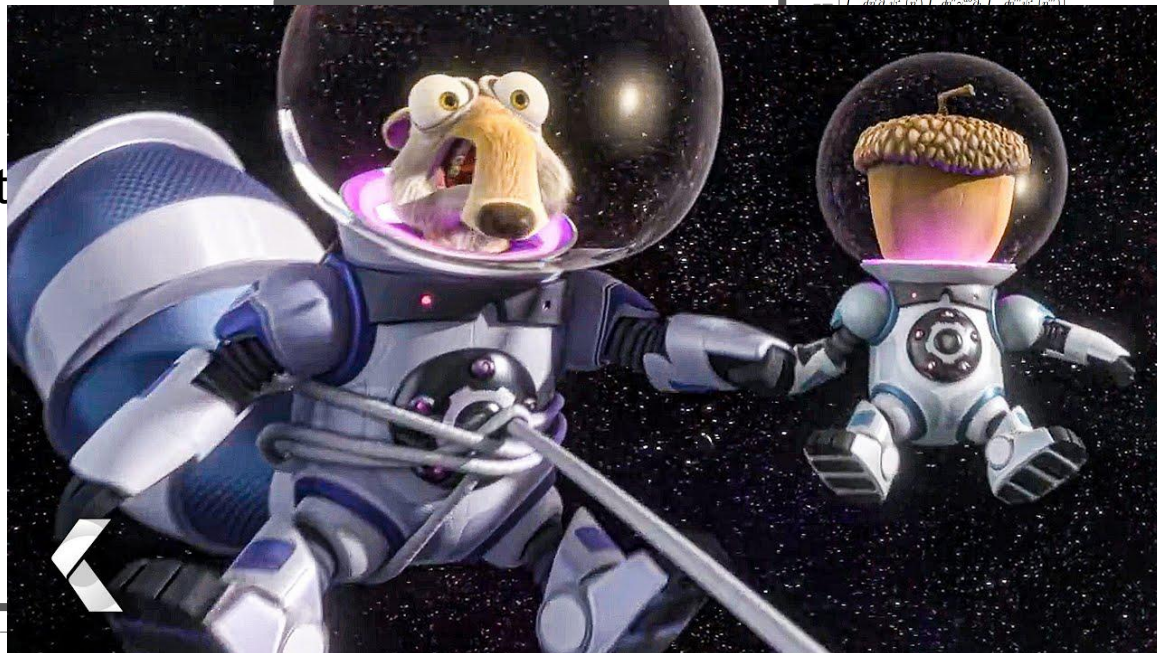
$$\begin{aligned}
 & + 2 \partial_\eta \left[ -v_s^I - \frac{1}{\mathcal{H}_s} (\partial_\eta v_s^I + \partial_\eta v_{||s}) \right] \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \int_{\eta_s}^{\eta_0} d\eta'' \psi^I(\eta'') \\
 & - 2 \alpha v_{||s} \partial_\eta \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') + \frac{4}{r_s} \int_{\eta_s}^{\eta_0} d\eta' [\psi^I(\eta') (-v_s^I(\eta') - 2 \int_{\eta_s}^{\eta_0} d\eta'' \partial_\eta \psi^I(\eta'')) \\
 & + \partial_\eta^2 \psi^I(\eta') \int_{\eta_s}^{\eta_0} d\eta'' \psi^I(\eta'')] \partial_\eta \left( \int_{\eta_s}^{\eta_0} d\eta'' \psi^I(\eta'') \right) + 4 \partial_\eta v_s^I \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \int_{\eta_s}^{\eta_0} d\eta'' \psi^I(\eta'') \\
 & - \frac{8}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') \int_{\eta_s}^{\eta_0} d\eta'' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \int_{\eta_s}^{\eta_0} d\eta''' \psi^I(\eta''') \\
 & + 2 \partial_\eta \left( \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') \right) \left[ \frac{1}{\mathcal{H}_s} \partial_\eta \left[ -v_s^I - \frac{1}{\mathcal{H}_s} (\partial_\eta v_s^I + \partial_\eta v_{||s}) \right] - 3 \int_{\eta_s}^{\eta_0} d\eta'' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \psi^I(\eta'') \right.
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{AS} = & \left( -\frac{2}{\mathcal{H}_s r_s} - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \left\{ -v_{||s}^I + \frac{3}{2} (v_s^I)^2 - 3 v_s^I v_s^A - 4 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta (\psi^I \psi^A(\eta')) \right. \\
 & - 2 v_s^I \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') - 2 \partial_\eta v_s^I \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \int_{\eta_s}^{\eta_0} d\eta'' \psi^I(\eta'') + \left. \frac{1}{\mathcal{H}_s} \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \left[ \frac{3}{2} (v_s^I)^2 \right. \right. \\
 & \left. \left. + \frac{1}{\mathcal{H}_s r_s} \left( 1 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s} + \frac{1}{\mathcal{H}_s r_s} \right) \left[ (v_s^I)^2 + 2 v_s^I \left( -v_{||s} - v_s^I - 2 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') \right) \right] \right. \right. \\
 & \left. \left. + 2 \left( 1 - \frac{1}{\mathcal{H}_s r_s} \right) \left[ 3 v_{||s}^I v_{||s} - \frac{1}{\mathcal{H}_s} v_{||s} \partial_\eta v_s^I + 2 \frac{\mathcal{H}'_s}{\mathcal{H}_s} v_{||s} \right. \right. \right. \\
 & \left. \left. - v_s^I \left[ -\frac{2}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') - 4 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') + \frac{2}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 \psi^I(\eta') \right] \right. \right. \\
 & \left. \left. - v_s^I \left[ -\frac{2}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') - 4 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') + \frac{2}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 \psi^I(\eta') \right] \right. \right. \\
 & \left. \left. + \left[ \frac{1}{\mathcal{H}_s^2} (\partial_\eta^2 v_s^I - \partial_\eta^2 v_s^I - \partial_\eta \partial_\eta v_s^I) + \left( 5 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} \partial_\eta v_s^I - \frac{3}{\mathcal{H}_s} \partial_\eta v_s^I \right. \right. \right. \\
 & \left. \left. - \left( 6 + 4 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) v_{||s} + \left[ \frac{1}{\mathcal{H}_s} \partial_\eta v_{||s} - \left( 8 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \frac{1}{\mathcal{H}_s} \partial_\eta v_{||s} \right] v_s^I + \frac{2}{\mathcal{H}_s^2} \partial_\eta v_s^I \partial_\eta v_{||s} \right. \right. \\
 & \left. \left. + \frac{1}{\mathcal{H}_s} \alpha v_{||s} \partial_\eta v_s^I + \left[ \frac{2}{\mathcal{H}_s} \partial_\eta v_s^I + \left( 10 + 2 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^I \right] \frac{1}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 \psi^I(\eta') \right. \right. \\
 & \left. \left. + \frac{2}{\mathcal{H}_s} \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') + \left[ \frac{2}{\mathcal{H}_s} \partial_\eta v_s^I + 2 \left( 4 + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) v_s^I \right] \frac{1}{r_s} \right. \right. \\
 & \left. \left. + \frac{1}{\mathcal{H}_s} \partial_\eta v_s^I + \left( 3 + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \partial_\eta v_s^I \right] \left( -2 \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') \right) \right. \\
 & \left. + \left[ \frac{1}{\mathcal{H}_s} (\partial_\eta^2 v_s^I - \partial_\eta^2 v_s^I - \partial_\eta \partial_\eta v_s^I) - \left( 6 + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} \partial_\eta v_s^I + \frac{3}{\mathcal{H}_s} \partial_\eta v_s^I \right. \right. \\
 & \left. \left. + 3 \left( 2 + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) v_s^I \left( -2 \int_{\eta_s}^{\eta_0} d\eta' \partial_\eta \psi^I(\eta') + \left( \frac{15}{2} + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) (v_s^I)^2 + \frac{1}{\mathcal{H}_s} (\partial_\eta v_s^I)^2 \right. \right. \right. \\
 & \left. \left. + \frac{2}{\mathcal{H}_s} \partial_\eta v_s^I \partial_\eta v_s^I + \left[ \frac{1}{\mathcal{H}_s} (\partial_\eta^2 v_s^I - \partial_\eta^2 v_s^I - \partial_\eta \partial_\eta v_s^I) + \frac{2}{\mathcal{H}_s} (\partial_\eta^2 v_s^I - \partial_\eta^2 v_s^I - \partial_\eta \partial_\eta v_s^I) \right. \right. \right. \\
 & \left. \left. + \left( 2 - 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} v_s^I + \left( 4 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} v_s^I \left( -7 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) v_s^I \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & + 6 \partial_\eta \psi_s^A \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \int_{\eta_s}^{\eta_0} d\eta'' \psi^I(\eta'') + \frac{2}{\mathcal{H}_s} \partial_\eta v_s^A \gamma_0^{\delta \delta^{(1)}} \int_{\eta_s}^{\eta_0} d\eta' \psi^I(\eta') \\
 & - \frac{2}{\mathcal{H}_s} \partial_\eta (\partial_\eta \psi_s^A) \int_{\eta_s}^{\eta_0} d\eta' \gamma_0^{\delta \delta^{(1)}} \partial_\eta \int_{\eta_s}^{\eta_0} d\eta'' \psi^I(\eta'') + \frac{8}{r_s} \int_{\eta_s}^{\eta_0} d\eta' (\psi^I \psi^A)(\eta') \\
 & - \frac{4}{r_s} \int_{\eta_s}^{\eta_0} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 (\psi^I \psi^A)(\eta') + \left[ \left( -\frac{2}{\mathcal{H}_s r_s} - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^A + \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^A \right] \delta^{(1)} \\
 & + \left[ \frac{1}{\mathcal{H}_s} \partial_\eta (\delta \delta^{(1)}) - \partial_\eta \delta^{(1)} \right] \frac{1}{\mathcal{H}_s} \psi_s^A. \quad (4.44)
 \end{aligned}$$

# Relativistic matter bispectrum of cosmic structure

- Why?
  - LCDM says it is
  - it is annoying
- How?
  - Computation



$$\begin{aligned}
 &+2\partial_\alpha \left[ -\psi_s^I - \frac{1}{\mathcal{H}_s} (\partial_\alpha \psi_s^I + \partial_\alpha v_{1s}) \right] \int_{\eta_0}^{\eta} d\eta' \gamma_0^{\alpha\beta} \partial_\beta \int_{\eta'}^{\eta} d\eta'' \psi^I(\eta'') \\
 &-2\alpha_{\text{eff}}^I \partial_\alpha \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') + \frac{4}{r_s} \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') \left( -\psi^I(\eta) - 2 \int_{\eta_0}^{\eta} d\eta'' \partial_\alpha v_{\alpha} \psi^I(\eta'') \right) \\
 &+ \gamma_0^{\alpha\beta} \partial_\alpha \left( \int_{\eta_0}^{\eta} d\eta' \psi_s^I(\eta') \right) \partial_\beta \left( \int_{\eta_0}^{\eta} d\eta' \psi_s^I(\eta') \right) + 4\partial_\alpha \psi_s^I \int_{\eta_0}^{\eta} d\eta' \gamma_0^{\alpha\beta} \partial_\beta \int_{\eta'}^{\eta} d\eta'' \psi^I(\eta'') \\
 &- 8 \left[ \int_{\eta_0}^{\eta} d\eta' \psi_s^I(\eta') \int_{\eta_0}^{\eta} d\eta'' \psi_s^I(\eta'') \int_{\eta_0}^{\eta} d\eta''' \psi^I(\eta''') \right]
 \end{aligned}$$

$$\begin{aligned}
 &+ \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^A \left[ -(\psi_s^I - \psi_s^A) - 2 \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') \right] + \frac{1}{\mathcal{H}_s} \psi_s^A \partial_\eta \psi_s^I \\
 &+ \left[ -\frac{1}{\mathcal{H}_s^2} (\partial_\eta^2 \psi_s^A - \partial_\eta^2 \psi_s^A - \partial_\eta \partial_t \psi_s^A) + \left( 5 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^A - \frac{3}{\mathcal{H}_s} \partial_t \psi_s^A \right. \\
 &\left. - \left( 6 + 4 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^A \right] v_{1s} + \left[ -\frac{1}{\mathcal{H}_s^2} \partial_\eta^2 v_{1s} - \left( 8 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \frac{1}{\mathcal{H}_s} \partial_t v_{1s} \right] \psi_s^A + \frac{2}{\mathcal{H}_s^2} \partial_\eta \psi_s^A \partial_t v_{1s} \\
 &+ \frac{1}{\mathcal{H}_s} \alpha_{\text{eff}}^I \partial_\alpha \psi_s^A + \left[ -\frac{2}{\mathcal{H}_s} \partial_\eta \psi_s^A + \left( 10 + 2 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^A \right] \frac{1}{r_s} \int_{\eta_0}^{\eta} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 \psi^I(\eta') \\
 &+ \frac{2}{\mathcal{H}_s} \psi_s^A \int_{\eta_0}^{\eta} d\eta' \Delta_2 \psi^I(\eta') + \left\{ \left[ -\frac{2}{\mathcal{H}_s} \partial_\eta \psi_s^A + 2 \left( 4 + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^A \right] \frac{1}{\mathcal{H}_s} \right.
 \end{aligned}$$

$$\begin{aligned}
 &+ \frac{4}{\mathcal{H}_s r_s} \psi_s^I - \frac{2}{\mathcal{H}_s^2} \int_{\eta_0}^{\eta} d\eta' \Delta_2 \psi^I(\eta') v_{1s} + \left[ 2 \left( 2 + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \partial_\eta v_{1s} \right. \\
 &\left. + \frac{2}{\mathcal{H}_s^2} \partial_\eta^2 v_{1s} \right] \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') + \left[ \frac{2}{\mathcal{H}_s} \left( 5 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \partial_\eta v_{1s} + \frac{2}{\mathcal{H}_s^2} \partial_\eta^2 v_{1s} \right] \int_{\eta_0}^{\eta} d\eta' \partial_\alpha \psi^I(\eta') \\
 &+ \frac{2}{\mathcal{H}_s} \alpha_{\text{eff}}^I \frac{1}{r_s} \int_{\eta_0}^{\eta} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 \psi^I(\eta') + \frac{2}{\mathcal{H}_s^2} \partial_\eta \psi_s^I \partial_\eta v_{1s} + \frac{1}{\mathcal{H}_s} \alpha_{\text{eff}}^I \partial_\alpha \psi_s^I \\
 &+ \left( 6 + 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \partial_\eta v_{1s} \left[ \psi_s^I - \frac{2}{\mathcal{H}_s} \alpha_{\text{eff}}^I \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') + \frac{1}{\mathcal{H}_s} \alpha_{\text{eff}}^I \partial_\alpha \psi_s^I \right. \\
 &\left. - \frac{6}{\mathcal{H}_s} \alpha_{\text{eff}}^I \partial_\alpha v_{1s} \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') + \left( \frac{4}{r_s} \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') \right)^2 + \left\{ \left[ 2 \left( -2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^I \right. \right. \right. \\
 &\left. \left. + \left( -4 - 2 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \int_{\eta_0}^{\eta} d\eta' \partial_\alpha \psi^I(\eta') - \frac{2}{\mathcal{H}_s} \partial_\eta \psi_s^I \right] \frac{1}{r_s} + 2 \left( -2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \int_{\eta_0}^{\eta} d\eta' \partial_\alpha \psi^I(\eta') \right. \\
 &\left. + 2 \left( -2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \partial_\eta \psi_s^I + \left( -1 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \partial_\eta \psi_s^I - \frac{1}{\mathcal{H}_s} \partial_\eta \partial_\alpha \psi_s^I \right] \left( -2 \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') \right) \\
 &+ \left[ \frac{3}{\mathcal{H}_s} \left( -1 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{3} \frac{1}{\mathcal{H}_s r_s} \right) \partial_\eta \psi_s^I + \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^I - \left( 2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^I \right] \psi_s^I
 \end{aligned}$$

$$\left[ \frac{4}{\mathcal{H}_s} \left( 2 - 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} \psi_s^I + \left( 4 - 3 \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s} \right) \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^I - \left( 7 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^I \right] \psi_s^I$$

$$\begin{aligned}
 &+ 6 \partial_\alpha \psi_s^A \int_{\eta_0}^{\eta} d\eta' \gamma_0^{\alpha\beta} \partial_\beta \int_{\eta'}^{\eta} d\eta'' \psi^I(\eta'') + \frac{2}{\mathcal{H}_s} \partial_\alpha \psi_s^A \partial_\beta \int_{\eta_0}^{\eta} d\eta' \psi^I(\eta') \\
 &- \frac{2}{\mathcal{H}_s} \partial_\alpha (\partial_\beta \psi_s^A) \int_{\eta_0}^{\eta} d\eta' \gamma_0^{\alpha\beta} \partial_\beta \int_{\eta'}^{\eta} d\eta'' \psi^I(\eta'') + \frac{8}{r_s} \int_{\eta_0}^{\eta} d\eta' (\psi^I \psi_s^A)(\eta') \\
 &- \frac{4}{r_s} \int_{\eta_0}^{\eta} d\eta' \frac{\eta' - \eta_0}{\eta_0 - \eta'} \Delta_2 (\psi^I \psi_s^A)(\eta') + \left[ \left( -\frac{2}{\mathcal{H}_s r_s} - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} \right) \psi_s^A + \frac{1}{\mathcal{H}_s} \partial_\eta \psi_s^A \right] \delta^{(1)} \\
 &+ \left[ \frac{1}{\mathcal{H}_s} \partial_\eta (\delta^{(1)} \psi_s^A) - \partial_\eta \delta^{(1)} \right] \frac{1}{\mathcal{H}_s} \psi_s^A.
 \end{aligned} \tag{4.44}$$

# Relativistic matter bispectrum of cosmic structure

- Why?
  - LCDM says it
  - it is annoying for inflation
- How?
  - Computation is too hard...
  - **SIMULATION!**



# Simulation pipeline

Inflation

# Simulation pipeline

Inflaton  $\phi$



Inflation



# Simulation pipeline

Inflaton  $\phi$

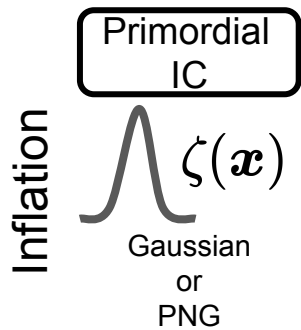


some other fancy field

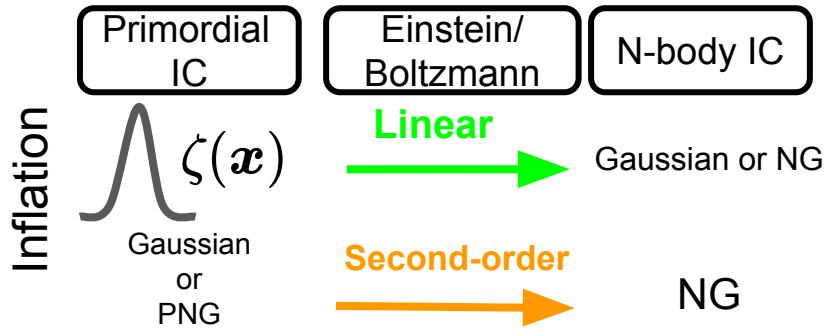


Inflation

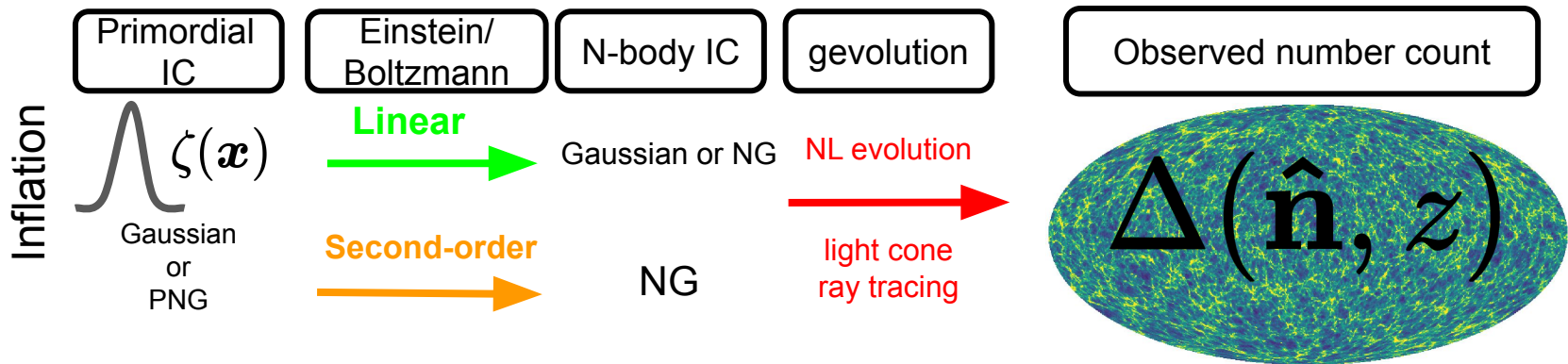
# Simulation pipeline



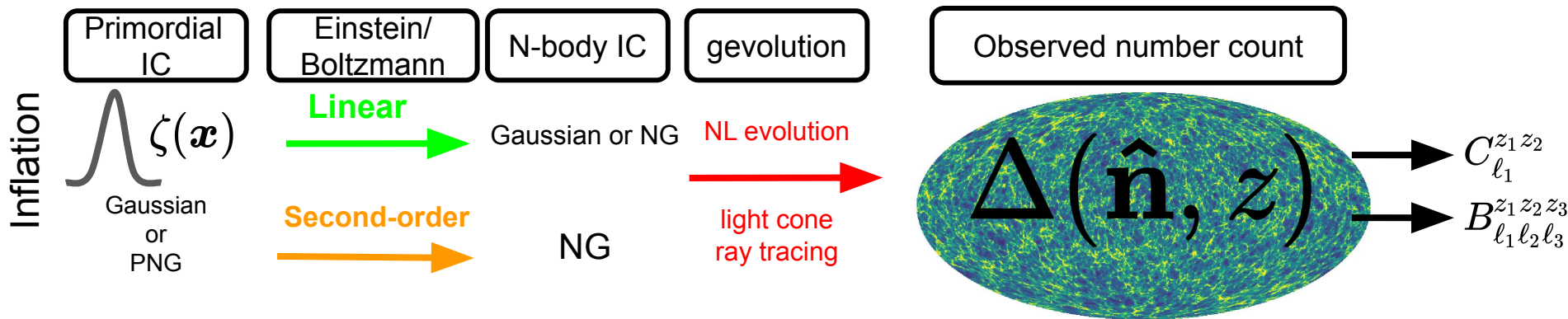
# Simulation pipeline



# Simulation pipeline

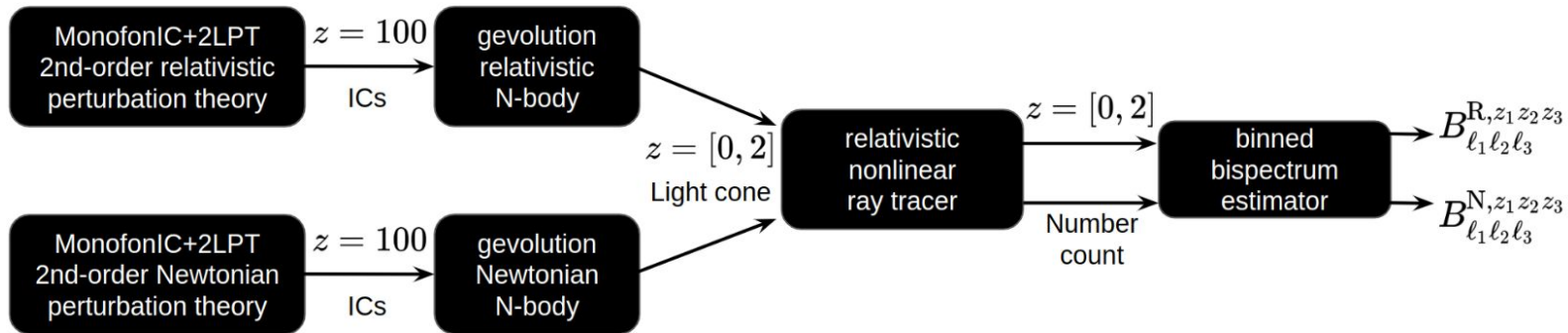
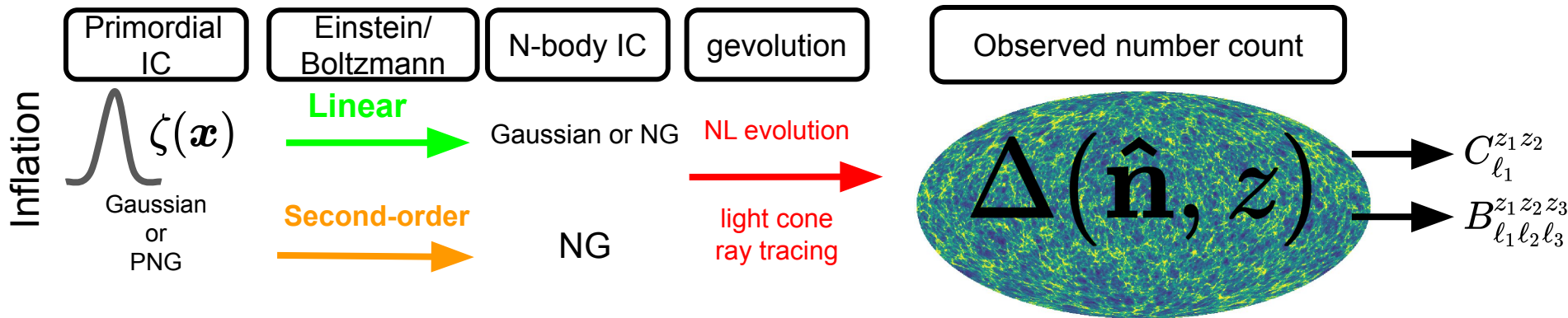


# Simulation pipeline

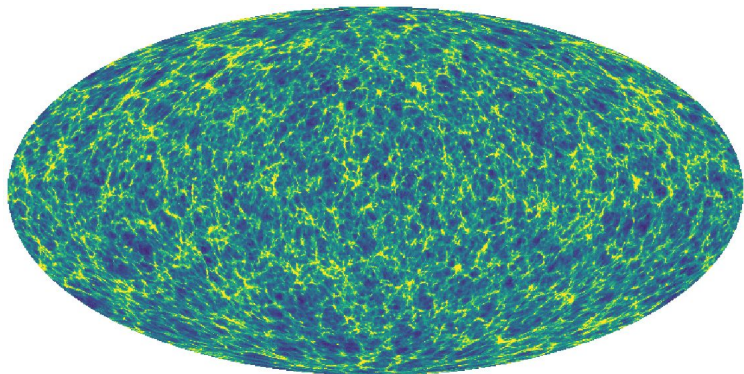


**IMPORTANT: Gauge invariance!**

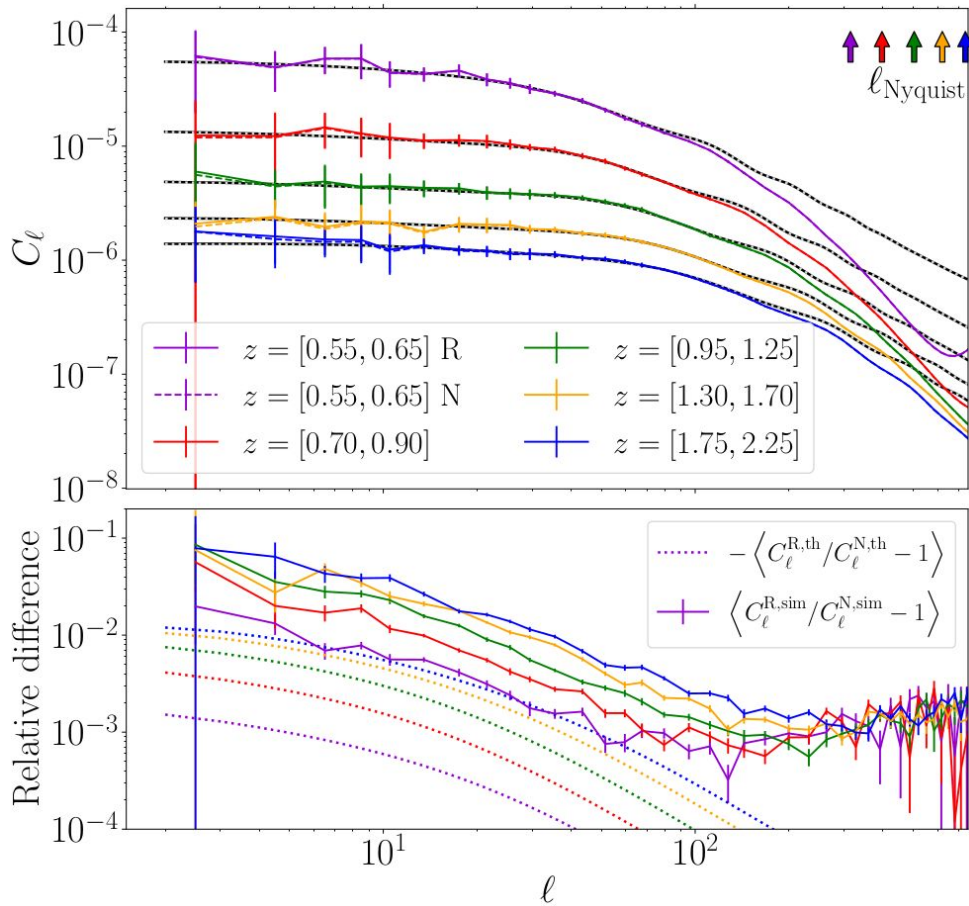
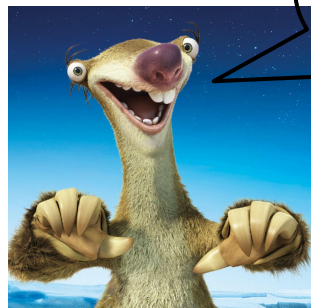
# Simulation pipeline



# Results



Nonlinear GR/Rad effects  
affect the power spectrum!  
z=2: 1-10% effect



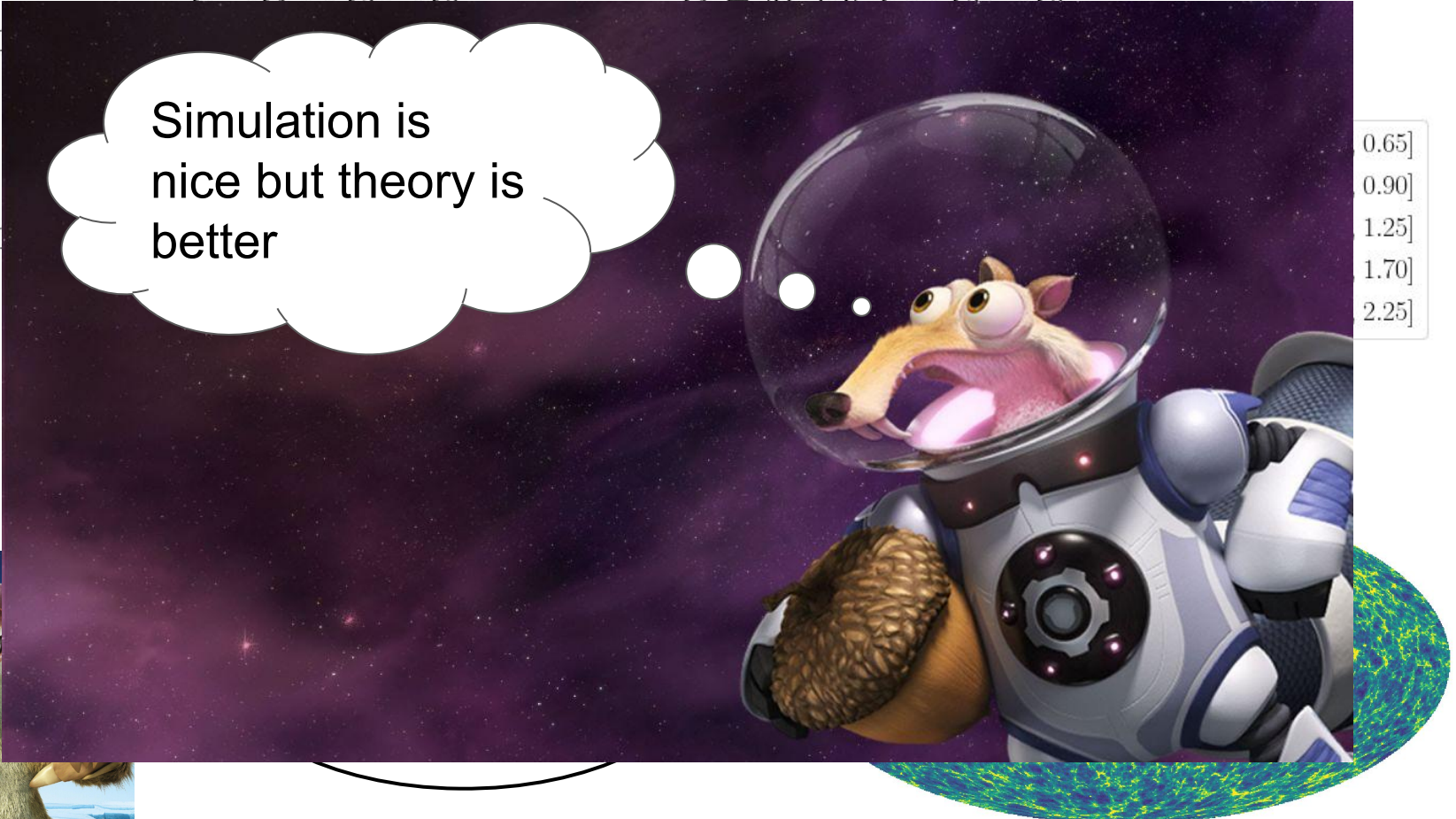
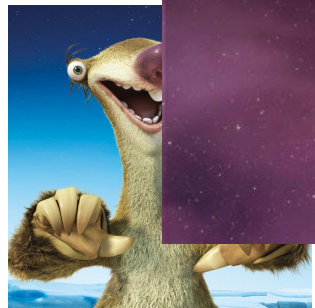
# Results

$$100 \times S[\langle B^{R-N} \rangle] / S[\langle B^N \rangle]$$

Simulation is nice but theory is better

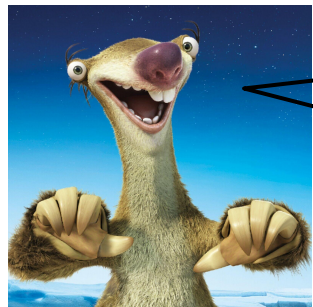
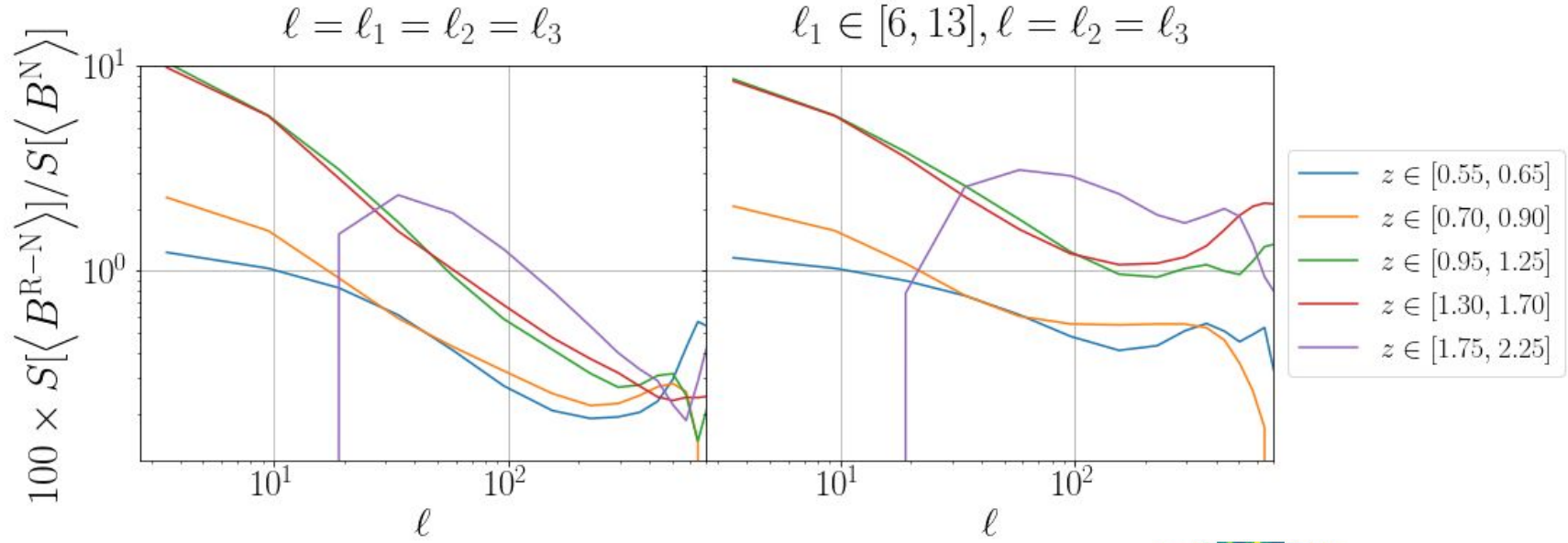
$$l = l_1 = l_2 = l_3 \quad l_1 \in [6, 13] \quad l = l_2 = l_3$$

- 0.65]
- 0.90]
- 1.25]
- 1.70]
- 2.25]

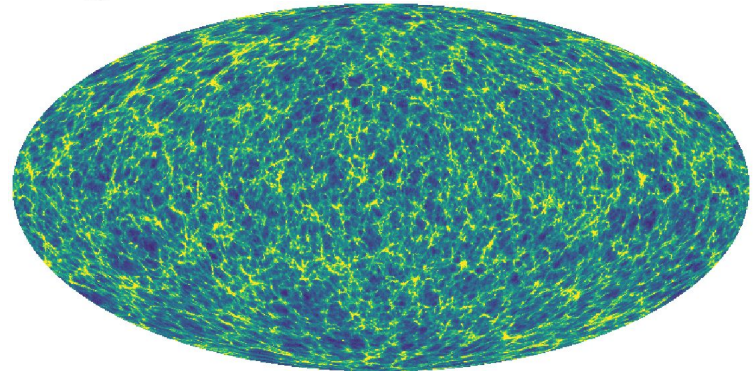




# Results



GR/Rad effects affect the bispectrum!  
 $z=2$ : 1-10% effect



## Angular bispectrum calculation

- Redshift bins
- GR up to  $\mathcal{H}^4/k^4$
- Radiation up to  $\mathcal{H}^4/k^4$
- Projection effects up to  $\mathcal{H}/k$

$$b_{\ell_1 \ell_2 \ell_3}^{r_1 r_2 r_3} = \frac{8}{\pi^3} \int dr'_1 dr'_2 dr'_3 W(r_1, r'_1) W(r_2, r'_2) W(r_3, r'_3) \\ \int dk_1 dk_2 dk_3 d\chi (k_1 k_2 k_3 \chi)^2 B^{r'_1 r'_2 r'_3}(k_1, k_2, k_3) \\ j_{\ell_1}(k_1 r'_1) j_{\ell_2}(k_2 r'_2) j_{\ell_3}(k_3 r'_3) j_{\ell_1}(k_1 \chi) j_{\ell_2}(k_2 \chi) j_{\ell_3}(k_3 \chi)$$

# Angular bispectrum calculation

- Redshift bins
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$$b_{\ell_1 \ell_2 \ell_3}^{\hat{z}_1 \hat{z}_2 \hat{z}_3} = b_{\ell_1 \ell_2 \ell_3}^{\delta_2} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r^2 v_2} + b_{\ell_1 \ell_2 \ell_3}^{(\partial_r^2 v_1)^2} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \partial_r^3 v_1} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \partial_r \delta_1} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r^2 v_1 \delta_1} \\ + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_2} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \partial_r^2 v_1} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \delta_2} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \delta} + b_{\ell_1 \ell_2 \ell_3}^{v^a \partial_a \partial_r v_1} + b_{\ell_1 \ell_2 \ell_3}^{\psi_1 \partial_r^3 v_1} + b_{\ell_1 \ell_2 \ell_3}^{\psi_1 \partial_r \delta_1} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \partial_r^2 \psi_1}, \quad (3.5)$$

# Angular bispectrum calculation

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- GR up to  $\mathcal{H}^4/k^4$
- Radiation up to  $\mathcal{H}^4/k^4$
- Projection effects up to  $\mathcal{H}/k$

$$b_{\ell_1 \ell_2 \ell_3}^{r_1 r_2 r_3} = \frac{8}{\pi^3} \int dr'_1 dr'_2 dr'_3 W(r_1, r'_1) W(r_2, r'_2) W(r_3, r'_3) \\ \int dk_1 dk_2 dk_3 d\chi (k_1 k_2 k_3 \chi)^2 B^{r'_1 r'_2 r'_3}(k_1, k_2, k_3) \\ j_{\ell_1}(k_1 r'_1) j_{\ell_2}(k_2 r'_2) j_{\ell_3}(k_3 r'_3) j_{\ell_1}(k_1 \chi) j_{\ell_2}(k_2 \chi) j_{\ell_3}(k_3 \chi)$$

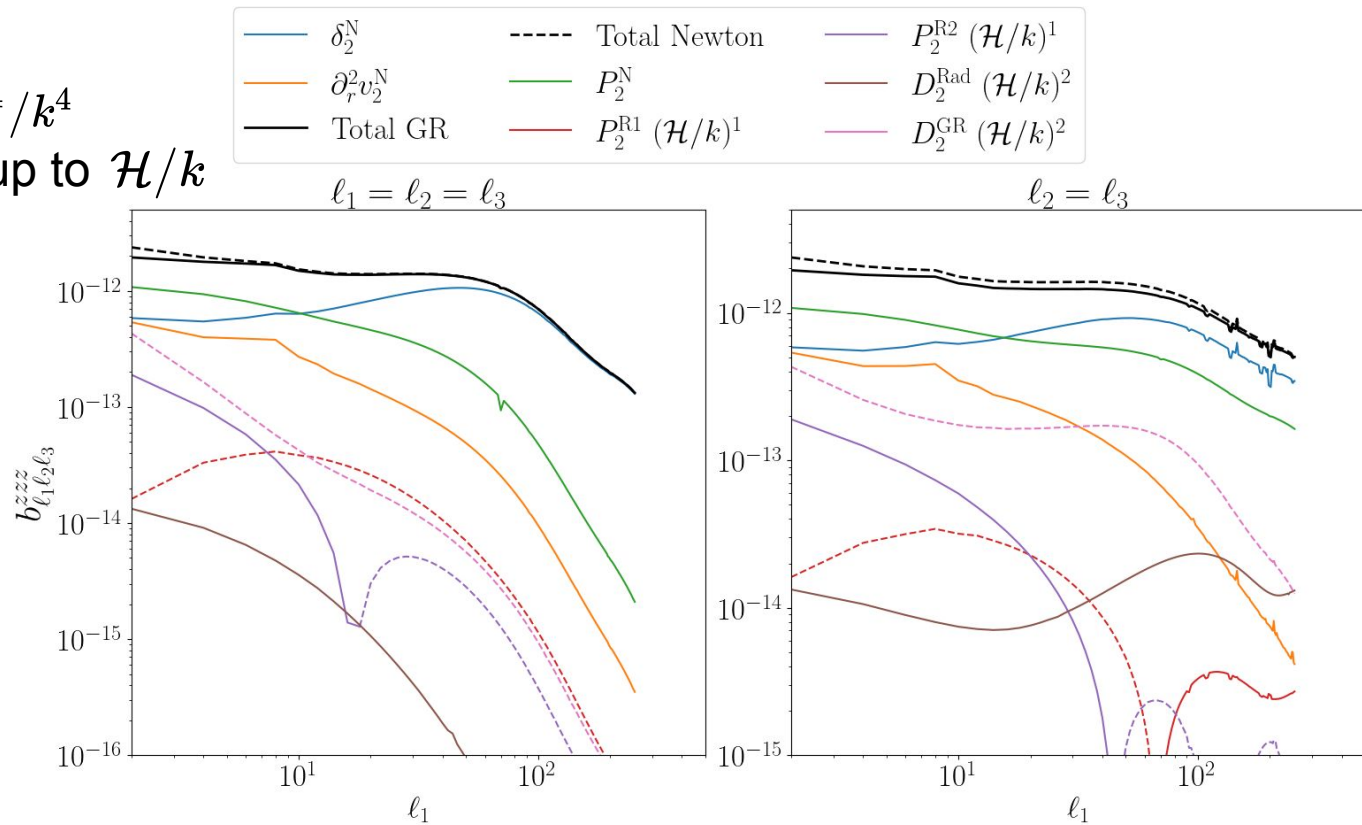
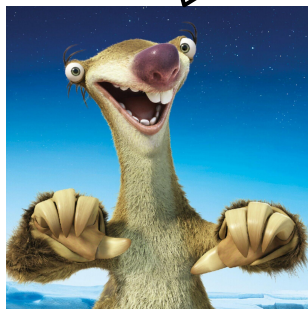
$$b_{\ell_1 \ell_2 \ell_3}^{\hat{z}_1 \hat{z}_2 \hat{z}_3} = b_{\ell_1 \ell_2 \ell_3}^{\delta_2} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r^2 v_2} + b_{\ell_1 \ell_2 \ell_3}^{(\partial_r^2 v_1)^2} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \partial_r^3 v_1} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \partial_r \delta_1} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r^2 v_1 \delta_1} \\ + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_2} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \partial_r^2 v_1} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \delta_2} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \delta} + b_{\ell_1 \ell_2 \ell_3}^{v^a \partial_a \partial_r v_1} + b_{\ell_1 \ell_2 \ell_3}^{\psi_1 \partial_r^3 v_1} + b_{\ell_1 \ell_2 \ell_3}^{\psi_1 \partial_r \delta_1} + b_{\ell_1 \ell_2 \ell_3}^{\partial_r v_1 \partial_r^2 \psi_1}, \quad (3.5)$$

$$b_{\ell_1 \ell_2 \ell_3}^{\delta_2} = 2 \sum_{mn} \int d\chi C_{\ell_2}^{(n,0,0)}(\chi) C_{\ell_3}^{(m,0,0)}(\chi) \\ \left( \chi^2 \int \frac{dr_1}{r_1^2} D^2(r_1) W(r_1) \left[ f_{nm}^{(-4)} I_{\ell_1}(-1, r_1, \chi) + f_{nm}^{(-2)} A_{\ell_1}(r_1, \chi) \right. \right. \\ \left. \left. + \sum_p c_p \left( f_{nm}^{(-4,R)} I_{\ell_1}(b + ip\eta - 1, r_1, \chi) + f_{nm}^{(-2,R)} I_{\ell_1}(b + ip\eta + 1, r_1, \chi) \right) \right] \right) \\ + f_{nm}^{(0)} D^2(\chi) W(\chi) + \mathcal{D}_{\ell_1} \left[ f_{nm}^{(2)} D^2(\chi) W(\chi) \right] + \mathcal{D}_{\ell_1}^2 \left[ f_{nm}^{(4)} D^2(\chi) W(\chi) \right] \Big). \quad (3.14)$$

# Angular bispectrum calculation

- Redshift bins
- GR up to  $\mathcal{H}^4/k^4$
- Radiation up to  $\mathcal{H}^4/k^4$
- Projection effects up to  $\mathcal{H}/k$

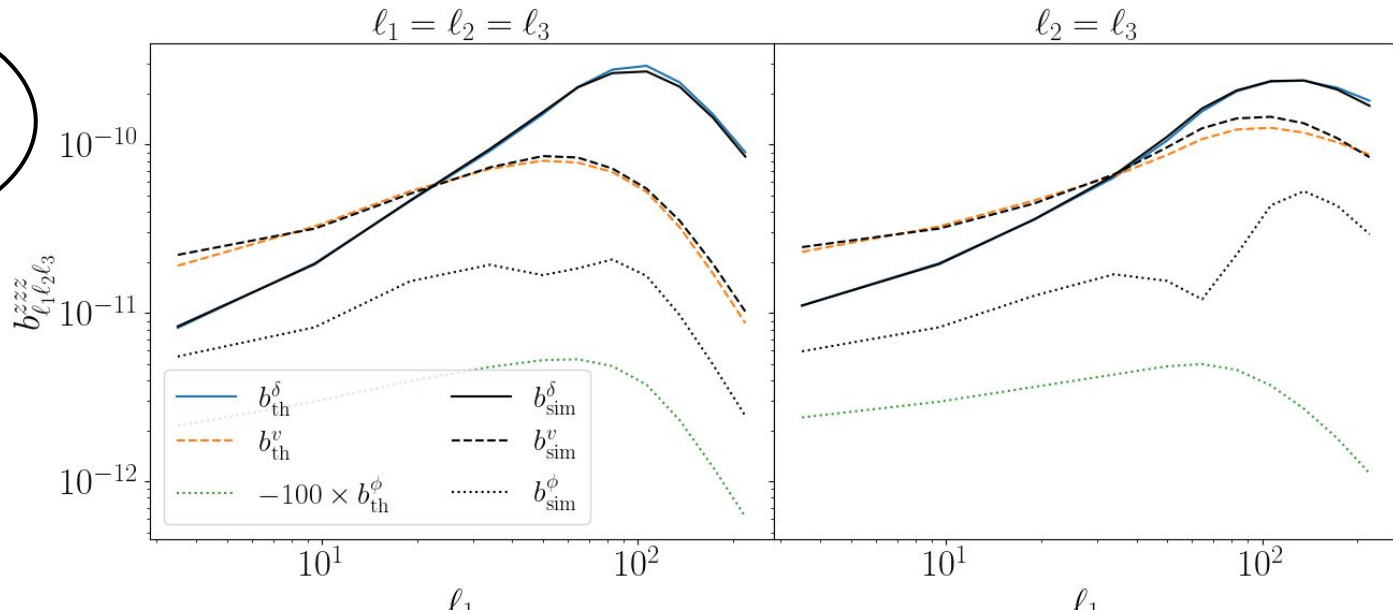
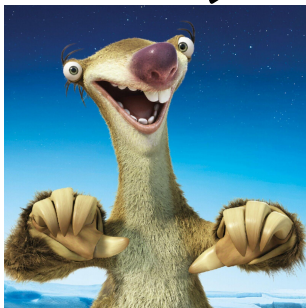
Preliminary



# Angular bispectrum calculation

- Redshift bins
- GR up to  $\mathcal{H}^4/k^4$
- Radiation up to  $\mathcal{H}^4/k^4$
- Projection effects up to  $\mathcal{H}/k$

Preliminary



# Angular bispectrum calculation

- Redshift bins
- GR up to  $\mathcal{H}^4/k^4$
- Radiation up to  $\mathcal{H}^4/k^4$
- Projection effects up to  $\mathcal{H}/k$

$$b_{\text{sim}}^{\text{GR}} \stackrel{?}{=} b_{\text{th}}^{\text{GR}} \quad \text{and} \quad b_{\text{sim}}^{\text{Rad}} \stackrel{?}{=} b_{\text{th}}^{\text{Rad}}$$

# Angular bispectrum calculation

- Redshift bins
- GR up to  $\mathcal{H}^4/k^4$
- Radiation up to  $\mathcal{H}^4/k^4$
- Projection effects up to  $\mathcal{H}/k$

$$b_{\text{sim}}^{\text{GR}} \stackrel{?}{=} b_{\text{th}}^{\text{GR}} \quad \text{and} \quad b_{\text{sim}}^{\text{Rad}} \stackrel{?}{=} b_{\text{th}}^{\text{Rad}}$$

Come back soon for the results, we are on it







$f_{NL}$

- GR effects are cool
- We have an accurate numerical model thanks to simulation
- Theory is hard as expected

$f_{NL}$

Thank you !

