

INFRARED INSENSITIVITY OF COSMOLOGICAL OBSERVABLES

MATTEO MAGI – University of Zurich

arXiv:2401.15143

arXiv:2306.09406

In collaboration with Jaiyul Yoo

- Relativistic Effects have a characteristic imprint

$$\delta_g \sim (\dots) \delta_m + (\dots) \boxed{\frac{\mathcal{H}}{k} \delta_m} + (\dots) \boxed{\frac{\mathcal{H}^2}{k^2} \delta_m}$$

Gravity on large scales
and initial conditions

- Relativistic Effects have a characteristic imprint

$$\delta_g \sim (\dots) \delta_m + (\dots) \boxed{\frac{\mathcal{H}}{k} \delta_m} + (\dots) \boxed{\frac{\mathcal{H}^2}{k^2} \delta_m}$$

Gravity on large scales
and initial conditions

Definition of the variance:

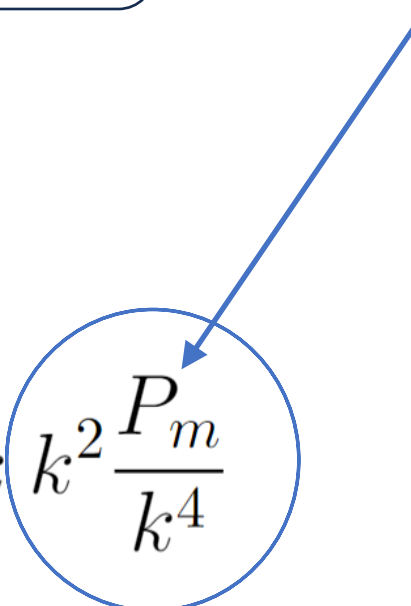
$$\langle \delta_g^2 \rangle := \int_0^\infty dk \, k^2 P_g(k)$$

- Relativistic Effects have a characteristic imprint

$$\delta_g \sim (\dots) \delta_m + (\dots) \boxed{\frac{\mathcal{H}}{k} \delta_m} + (\dots) \boxed{\frac{\mathcal{H}^2}{k^2} \delta_m}$$

Gravity on large scales
and initial conditions

Definition of the variance:

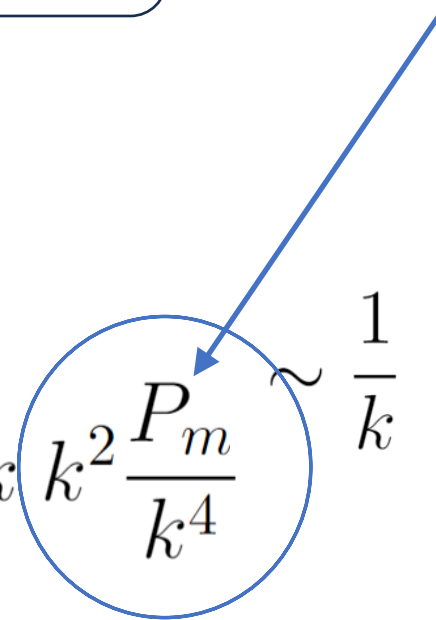
$$\langle \delta_g^2 \rangle := \int_0^\infty dk \, k^2 P_g(k) \ni \int_0^\infty dk \, \left(k^2 \frac{P_m}{k^4} \right)$$


- Relativistic Effects have a characteristic imprint

$$\delta_g \sim (\dots) \delta_m + (\dots) \boxed{\frac{\mathcal{H}}{k} \delta_m} + (\dots) \boxed{\frac{\mathcal{H}^2}{k^2} \delta_m}$$

Gravity on large scales
and initial conditions

Definition of the variance:

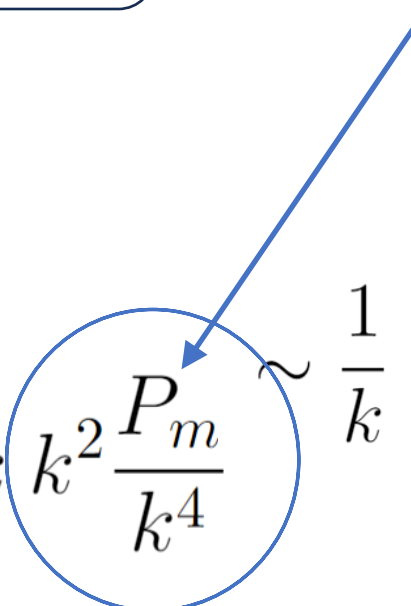
$$\langle \delta_g^2 \rangle := \int_0^\infty dk \, k^2 P_g(k) \ni \int_0^\infty dk \, \left(k^2 \frac{P_m}{k^4} \right) \sim \frac{1}{k} \text{ for } k \rightarrow 0$$


- Relativistic Effects have a characteristic imprint

$$\delta_g \sim (\dots) \delta_m + (\dots) \boxed{\frac{\mathcal{H}}{k} \delta_m} + (\dots) \boxed{\frac{\mathcal{H}^2}{k^2} \delta_m}$$

Gravity on large scales
and initial conditions

Definition of the variance:

$$\langle \delta_g^2 \rangle := \int_0^\infty dk \, k^2 P_g(k) \ni \int_0^\infty dk \, \left(k^2 \frac{P_m}{k^4} \right) \sim \frac{1}{k} \text{ for } k \rightarrow 0 = \infty$$


- Relativistic Effects have a characteristic imprint

$$\delta_g \sim (\dots) \delta_m + (\dots) \boxed{\frac{\mathcal{H}}{k} \delta_m} + (\dots) \boxed{\frac{\mathcal{H}^2}{k^2} \delta_m}$$

Gravity on large scales
and initial conditions

Definition of the variance:

$$\langle \delta_g^2 \rangle := \int_0^\infty dk \, k^2 P_g(k) \ni \int_0^\infty dk \, k^2 \frac{P_m}{k^4} \sim \frac{1}{k} \text{ for } k \rightarrow 0 = \infty$$



IR DIVERGENCE caused by fluctuations at $k=0$

Vast literature on IR Divergences

Biern, Yoo (2017)

Scaccabarozzi, Yoo, Biern (2018)

Grimm, Scaccabarozzi, Yoo, Biern, Gong (2020)

Castorina, Di Dio (2022)

Divergent terms cancel out in the limit $k \rightarrow 0$ in the full gauge-invariant expression for the observable

$$\langle \delta_g^2 \rangle := \int_0^\infty dk \, k^2 P_g(k) \sim k^3 \text{ for } k \rightarrow 0 < \infty$$

Fluctuations on large scales do not affect statistics =: **IR Insensitivity**

There are **2 assumptions** for these cancellations to happen

- Adiabatic conditions on large scales
 - General Relativity
- 
- Λ CDM model

\Rightarrow In **Λ CDM** observable statistics are IR Insensitive, **no IR Divergences!**

There are **2 assumptions** for these cancellations to happen

- Adiabatic conditions on large scales
 - General Relativity
- 
- Λ CDM model

\Rightarrow In **Λ CDM** observable statistics are IR Insensitive, **no IR Divergences!**

How much freedom do we have in the gravity sector if we want to avoid the IR Divergences?

What causes IR Divergence? General approach:

- Light follows null geodesics of the metric $g_{\mu\nu} = \text{FRW} + \text{linear perturbations}$
- Energy-momentum tensor is covariantly conserved
- No commitment to field equations

Look at specific observables and isolate the contributions of long-wavelength fluctuations by expanding in powers of $k\bar{r} \ll 1$

- Galaxy number density
- Luminosity distance
- CMB anisotropies

$$\begin{aligned} \delta g &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \delta D_L &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \Theta &\xrightarrow[k \rightarrow 0]{} (\dots) \end{aligned}$$

Contain

$$\Phi, \Psi, \nu, \delta_m, \delta_\gamma$$

• Galaxy number density	$\delta_g \xrightarrow[k \rightarrow 0]{} (\dots)$	Contain $\Phi, \Psi, \nu, \delta_m, \delta_\gamma$
• Luminosity distance	$\delta D_L \xrightarrow[k \rightarrow 0]{} (\dots)$	
• CMB anisotropies	$\Theta \xrightarrow[k \rightarrow 0]{} (\dots)$	

Sufficient conditions for $k \rightarrow 0$ that make any statistics finite:

$$\mathcal{R} = \zeta \qquad \frac{\delta \rho}{\bar{\rho}'} = \frac{\delta p}{\bar{p}'}$$

**Conditions for
IR Insensitivity**

- Galaxy number density
- Luminosity distance
- CMB anisotropies

$$\begin{aligned} \delta g &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \delta D_L &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \Theta &\xrightarrow[k \rightarrow 0]{} (\dots) \end{aligned}$$

Contain

$$\Phi, \Psi, \nu, \delta_m, \delta_\gamma$$

Sufficient conditions for $k \rightarrow 0$ that make any statistics finite:

$$\mathcal{R} = \zeta$$

$$\frac{\delta \rho}{\bar{\rho}'} = \frac{\delta p}{\bar{p}'}$$

Adiabaticity

**Conditions for
IR Insensitivity**

- Galaxy number density
- Luminosity distance
- CMB anisotropies

$$\begin{aligned} \delta g &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \delta D_L &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \Theta &\xrightarrow[k \rightarrow 0]{} (\dots) \end{aligned}$$

Contain

$$\Phi, \Psi, \nu, \delta_m, \delta_\gamma$$

Sufficient conditions for $k \rightarrow 0$ that make any statistics finite:

$$\mathcal{R} = \zeta$$

$$\frac{\delta \rho}{\bar{\rho}'} = \frac{\delta p}{\bar{p}'}$$

**Conditions for
IR Insensitivity**

Adiabaticity

\sim

Initial conditions

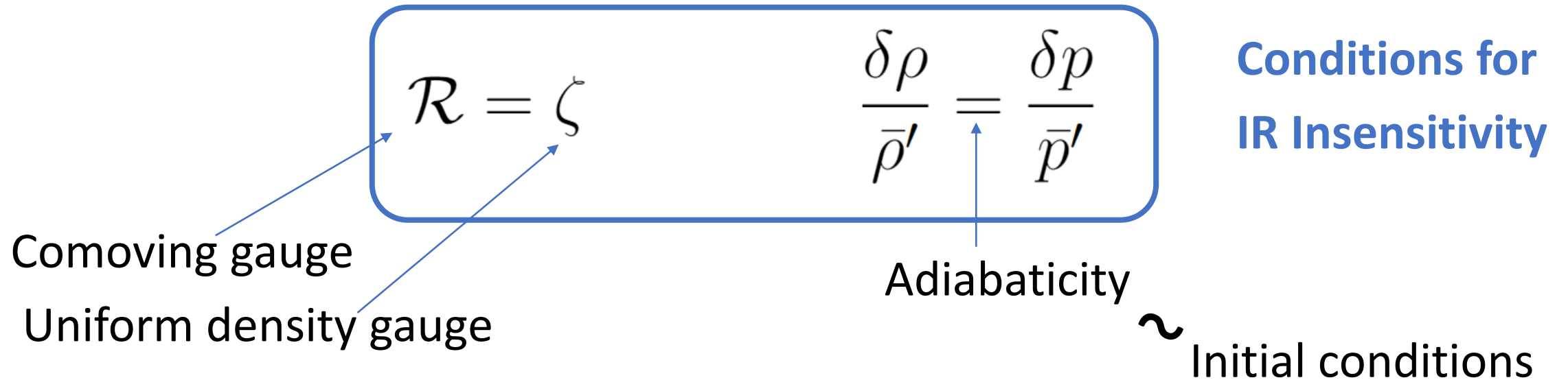
- Galaxy number density
- Luminosity distance
- CMB anisotropies

$$\begin{aligned} \delta g &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \delta D_L &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \Theta &\xrightarrow[k \rightarrow 0]{} (\dots) \end{aligned}$$

Contain

$$\Phi, \Psi, v, \delta_m, \delta_\gamma$$

Sufficient conditions for $k \rightarrow 0$ that make any statistics finite:



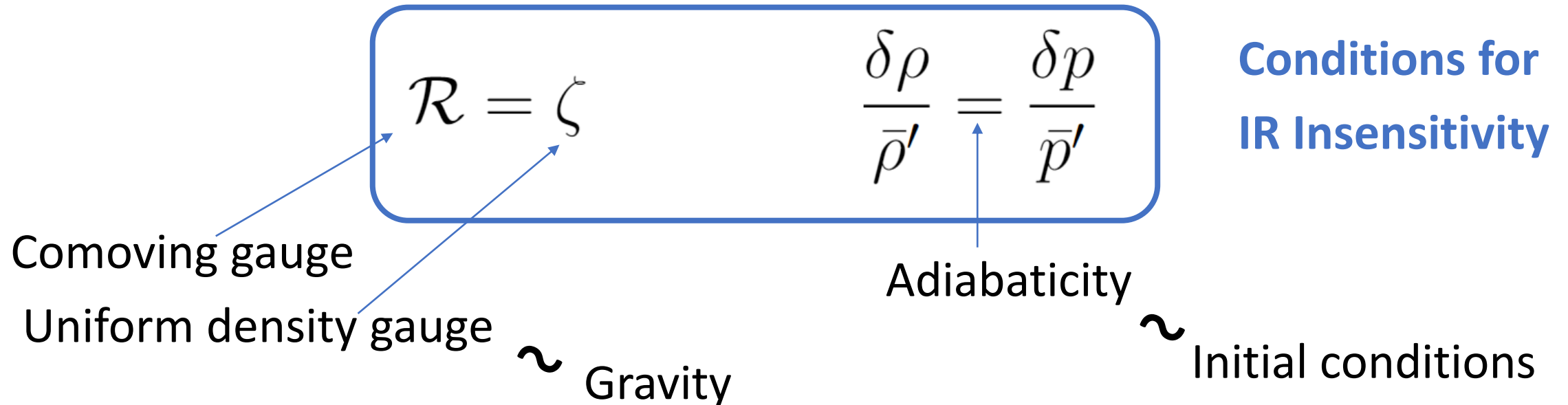
- Galaxy number density
- Luminosity distance
- CMB anisotropies

$$\begin{aligned} \delta g &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \delta D_L &\xrightarrow[k \rightarrow 0]{} (\dots) \\ \Theta &\xrightarrow[k \rightarrow 0]{} (\dots) \end{aligned}$$

Contain

$$\Phi, \Psi, v, \delta_m, \delta_\gamma$$

Sufficient conditions for $k \rightarrow 0$ that make any statistics finite:



Assume adiabaticity. Gravity theory leads to IR Insensitivity if field equations admit $\mathcal{R} = \zeta$

- **General Relativity**

Combine time-time and time-space Einstein equations for $k \rightarrow 0$

$$\Rightarrow \mathcal{R} = \zeta$$

- **Horndeski theory** $S_H = \int d^4x \sqrt{-g} \left(G_4(\phi) R + G_3(\phi, X) \square \phi + G_2(\phi, X) \right)$

More freedom than GR

Found both $\mathcal{R} = \zeta$ and $\mathcal{R} \neq \zeta$ solutions for $k \rightarrow 0$

Solution with $\mathcal{R} = \zeta$ has a peculiarity: **Weinberg mode in Horndeski theory**

$$\frac{\delta\rho}{\bar{\rho}'} = \frac{\delta p}{\bar{p}'} = \frac{\delta\phi}{\bar{\phi}'}$$

Exists if a generalized adiabatic condition is satisfied: Much stronger requirement than what inflation can provide

Take home message

- Large-scale adiabaticity is a fundamental requirement for IR Insensitivity, independently of gravity
- GR is not the only theory that allows for the cancellation of IR divergent terms
- Horndeski theory can provide the same cancellations for a specific solution of the field equations. This solution looks very similar to GR at the perturbative level

\Rightarrow General Relativity is a special theory

Summary

- Relativistic Effects introduce terms in the observables that taken individually lead to IR Divergences in the statistics
- Without committing to any field equations we found sufficient conditions that lead to the cancellations of these dangerous terms

$$\mathcal{R} = \zeta \qquad \frac{\delta \rho}{\bar{\rho}'} = \frac{\delta p}{\bar{p}'} \qquad \text{for } k \rightarrow 0$$

- We found Weinberg mode in Horndeski theory satisfies the conditions
- IR Divergences can be cured even if gravity is not GR but that requires fine-tuning

$$\begin{aligned}\delta D_{L,0} &= \left(\frac{1}{\bar{r}_z \mathcal{H}_z} - 1 \right) \left[\mathcal{R}_m + h_{\parallel\parallel\parallel} \right]_{\bar{0}}^z - \frac{3}{2} h_{\parallel\parallel\parallel}(\eta_{\bar{0}}) \\ &\quad - \frac{1}{2} h_{\parallel\parallel\parallel} + \mathcal{R}_m - \frac{1}{\bar{r}_z} \int_0^{\bar{r}_z} d\bar{r} \left(\mathcal{R}_m - 2h_{\parallel\parallel\parallel} \right) ,\end{aligned}$$

$$\delta_{g,0} = 3b \left(\zeta_m - \mathcal{R}_m \right) + e_z \left[\mathcal{R}_m + h_{\parallel\parallel\parallel} \right]_{\bar{0}}^z + \delta V_0 ,$$

$$\delta V_0 = 2\delta D_{L,0} - \left(1 - \frac{\mathcal{H}'_z}{\mathcal{H}_z^2} \right) \left[\mathcal{R}_m + h_{\parallel\parallel\parallel} \right]_{\bar{0}}^z - \frac{\mathcal{R}'_m + h'_{\parallel\parallel\parallel}}{\mathcal{H}_z} ,$$

$$\Theta_0 = \zeta_\gamma - \mathcal{R}_m(\eta_{\bar{0}}) + h_{\parallel\parallel\parallel} - h_{\parallel\parallel\parallel}(\eta_{\bar{0}})$$