INFRARED INSENSITIVITY OF COSMOLOGICAL OBSERVABLES

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In collaboration with Jaiyul Yoo

- Relativistic Effects have a characteristic imprint

$$\delta_g \sim (\cdots)\,\delta_m + (\cdots)\boxed{\frac{\mathcal{H}}{k}\delta_m} + (\cdots)\boxed{\frac{\mathcal{H}^2}{k^2}\delta_m}$$

Gravity on large scales
and initial conditions

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IR DIVERGENCE caused by fluctuations at k=0

Vast literature on IR Divergences

Biern, Yoo (2017)
Scaccabarozzi, Yoo, Biern (2018)
Grimm, Scaccabarozzi, Yoo, Biern, Gong (2020)
Castorina, Di Dio (2022)

Divergent terms cancel out in the limit k $\rightarrow$ 0 in the full gauge-invariant expression for the observable

$$\langle \delta_g^2 \rangle := \int_0^\infty dk\, k^2 P_g(k) < \infty$$

$\sim k^3$ for $k \rightarrow 0$

Fluctuations on large scales do not affect statistics =: **IR Insensitivity**

There are **2 assumptions** for these cancellations to happen

- Adiabatic conditions on large scales
- General Relativity

} ΛCDM model

$\Rightarrow$ In **ΛCDM** observable statistics are IR Insensitive, **no IR Divergences**!

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How much freedom do we have in the gravity sector if we want to avoid the IR Divergences?

What causes IR Divergence? General approach:

- Light follows null geodesics of the metric $g_{\mu\nu}$ = FRW+ linear perturbations
- Energy-momentum tensor is covariantly conserved
- No commitment to field equations

Look at specific observables and isolate the contributions of long-wavelength fluctuations by expanding in powers of $k\bar{r} \ll 1$

- Galaxy number density $\delta_g \xrightarrow[k\to 0]{} (\cdots)$
- Luminosity distance $\delta D_L \xrightarrow[k\to 0]{} (\cdots)$
- CMB anisotropies $\Theta \xrightarrow[k\to 0]{} (\cdots)$

Contain

$$\Phi,\ \Psi,\ v,\delta_m,\ \delta_\gamma$$

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Sufficient conditions for k $\longrightarrow$ 0 that make any statistics finite:

$$\mathcal{R} = \zeta \qquad \frac{\delta\rho}{\bar{\rho}'} = \frac{\delta p}{\bar{p}'}$$

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Conditions for IR Insensitivity

Comoving gauge

Uniform density gauge

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Conditions for IR Insensitivity

Comoving gauge

Uniform density gauge ~ Gravity

Adiabaticity ~ Initial conditions

Assume adiabaticity. Gravity theory leads to IR Insensitivity if field equations admit $\mathcal{R} = \zeta$

- **General Relativity**

Combine time-time and time-space Einstein equations for k $\to$ 0

$\Rightarrow \ \mathcal{R} = \zeta$

- **Horndeski theory** $S_H = \int d^4x\sqrt{-g}\Big(G_4(\phi)R + G_3(\phi, X)\Box\phi + G_2(\phi, X)\Big)$

More freedom than GR

Found both $\mathcal{R} = \zeta$ and $\mathcal{R} \neq \zeta$ solutions for k $\to$ 0

Solution with $\mathcal{R} = \zeta$ has a peculiarity: **Weinberg mode in Horndeski theory**

$$\frac{\delta\rho}{\bar{\rho}'} = \frac{\delta p}{\bar{p}'} = \frac{\delta\phi}{\bar{\phi}'}$$

Exists if a generalized adiabatic condition is satisfied: Much stronger requirement than what inflation can provide

Take home message

- Large-scale adiabaticity is a fundamental requirement for IR Insensitivity, independently of gravity
- GR is not the only theory that allows for the cancellation of IR divergent terms
- Horndeski theory can provide the same cancellations for a specific solution of the field equations. This solution looks very similar to GR at the perturbative level

$\Rightarrow$ General Relativity is a special theory

Summary

- Relativistic Effects introduce terms in the observables that taken individually lead to IR Divergences in the statistics

- Without committing to any field equations we found sufficient conditions that lead to the cancellations of these dangerous terms

$$\mathcal{R} = \zeta \qquad \frac{\delta\rho}{\bar{\rho}'} = \frac{\delta p}{\bar{p}'} \qquad \text{for} \quad k \to 0$$

- We found Weinberg mode in Horndeski theory satisfies the conditions

- IR Divergences can be cured even if gravity is not GR but that requires fine-tuning

$$
\begin{aligned}
\delta D_{L,0} &= \left(\frac{1}{\bar{r}_z \mathcal{H}_z} - 1\right)\left[\mathcal{R}_m + h_{\|\|}\right]_{\bar{o}}^{z} - \frac{3}{2} h_{\|\|}(\eta_{\bar{o}}) \\
&\quad - \frac{1}{2} h_{\|\|} + \mathcal{R}_m - \frac{1}{\bar{r}_z} \int_0^{\bar{r}_z} d\bar{r}\ \left(\mathcal{R}_m - 2h_{\|\|}\right) , \\
\delta_{g,0} &= 3b\left(\zeta_m - \mathcal{R}_m\right) + e_z\left[\mathcal{R}_m + h_{\|\|}\right]_{\bar{o}}^{z} + \delta V_0 , \\
\delta V_0 &= 2\delta D_{L,0} - \left(1 - \frac{\mathcal{H}'_z}{\mathcal{H}_z^2}\right)\left[\mathcal{R}_m + h_{\|\|}\right]_{\bar{o}}^{z} - \frac{\mathcal{R}'_m + h'_{\|\|}}{\mathcal{H}_z} , \\
\Theta_0 &= \zeta_\gamma - \mathcal{R}_m(\eta_{\bar{o}}) + h_{\|\|} - h_{\|\|}(\eta_{\bar{o}})
\end{aligned}
$$