Towards Rigorous Modeling of Counts-in-Cells Statistics

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Counts in Cells

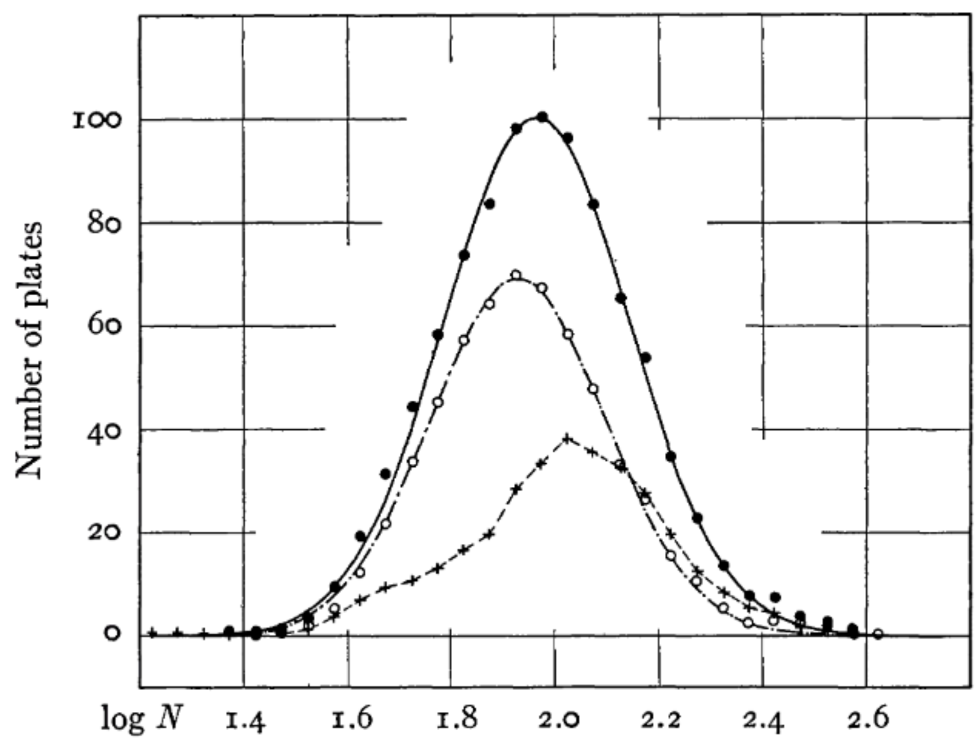


FIG. 8.—Frequency distribution of log N reduced to the galactic poles. Data from Table XIV. Crosses represent the 331 extra-survey fields; circles, the 587 survey fields; disks, the combined data, 918 fields. The smooth curves through the survey fields and the combined data (the two upper curves) are normal error-curves adjusted to the points.

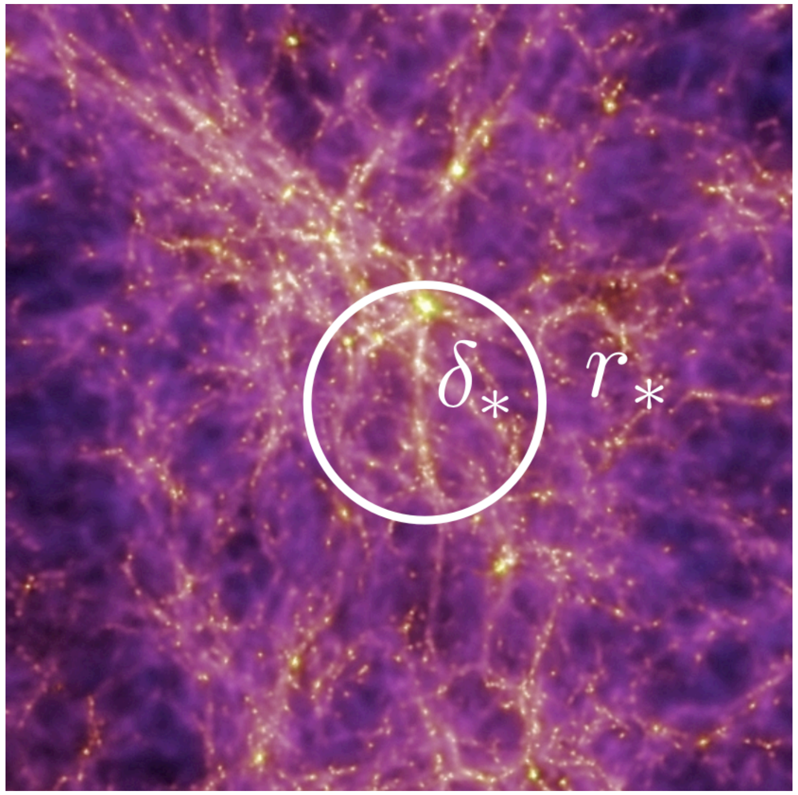
E. Hubble, 1934

$N_{gal} = 10$ $N_{gal} = 40$

Theoretical model from first principles?

Theory

$\mathcal{P}(\delta_*)$ - probability that a cell of radius r_* has averaged density contrast δ_*



$$\alpha \sim g(z)^2 \sigma_{r_*}^2 \ll 1 \qquad \sigma_{r_*}^2 = \langle \delta^2 \rangle_{r_*}$$

$$\mathcal{P}(\delta_*) = \exp\left\{ -\frac{1}{\alpha}(a_0 + \alpha a_1 + \ldots) \right\}$$

Saddle point solution
('instanton')
defined by spherical collapse
(Valageas'02)

Prefactor due to fluctuations
('determinant')
contains aspherical corrections
(M.M. Ivanov'19)

Theory

Spherical collapse:

$$\mathcal{P}(\delta_*) \propto \exp\left\{-\frac{F^2(\delta_*)}{2g^2\sigma^2_{R_*}}\right\}$$

$$F \equiv \delta_L$$

$$R_* = r_*(1+\delta_*)^{1/3}$$

Pre-exponential factor:

$$\delta(\mathbf{r}) = \delta_0(r)Y_0(\hat{\mathbf{r}}) + \sum_{\ell>0}\sum_{m=-\ell}^{\ell}\delta_{lm}(r)Y_{lm}(\hat{\mathbf{r}})$$

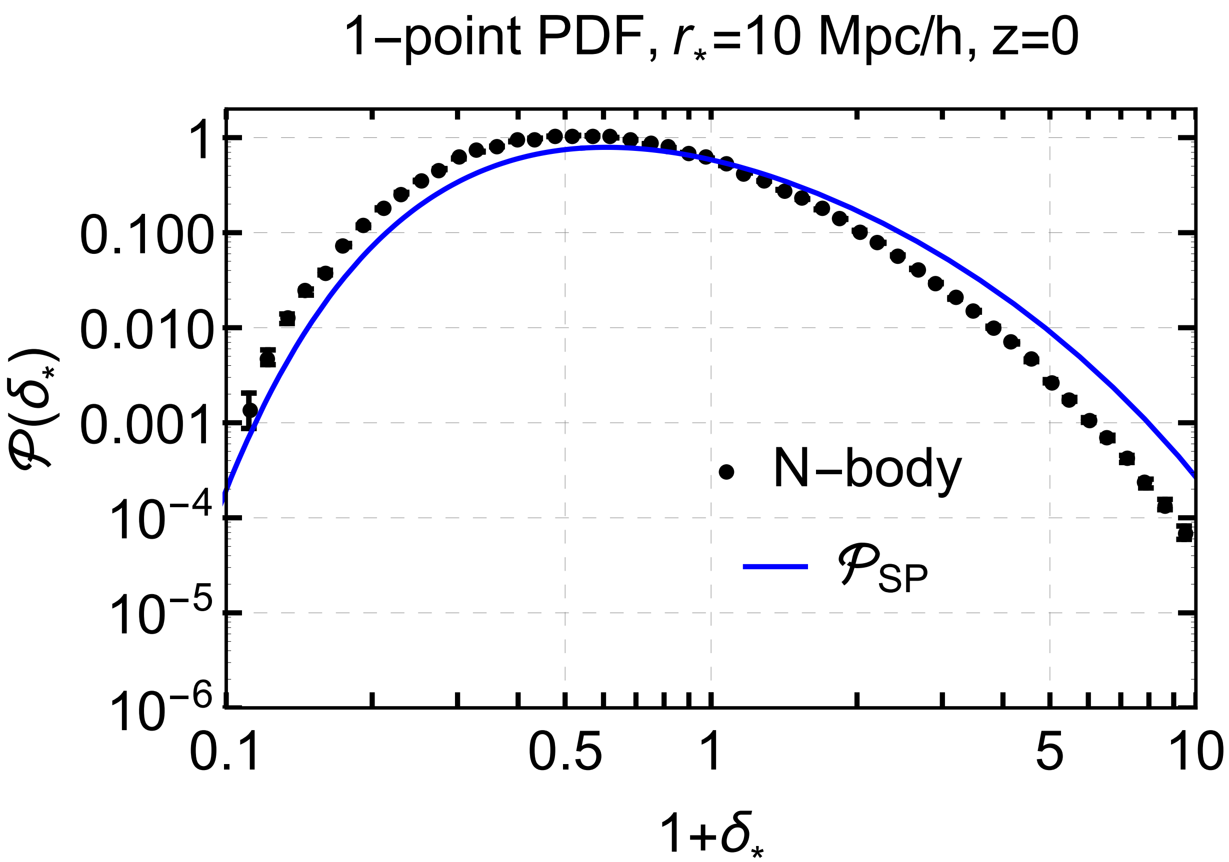
$$\mathcal{P} = \mathcal{P}_{SP} \times \mathcal{A}_{ASP}$$

$$\mathcal{P}(\delta_*) = \mathcal{A}_0 \cdot \prod_{\ell>0}\mathcal{A}_\ell(\delta_*) \cdot \exp\left\{-\frac{F^2(\delta_*)}{2g^2\sigma^2_{R_*}}\right\}$$

Monopole prefactor evaluated exactly

Aspherical prefactor can be calculated on grid numerically

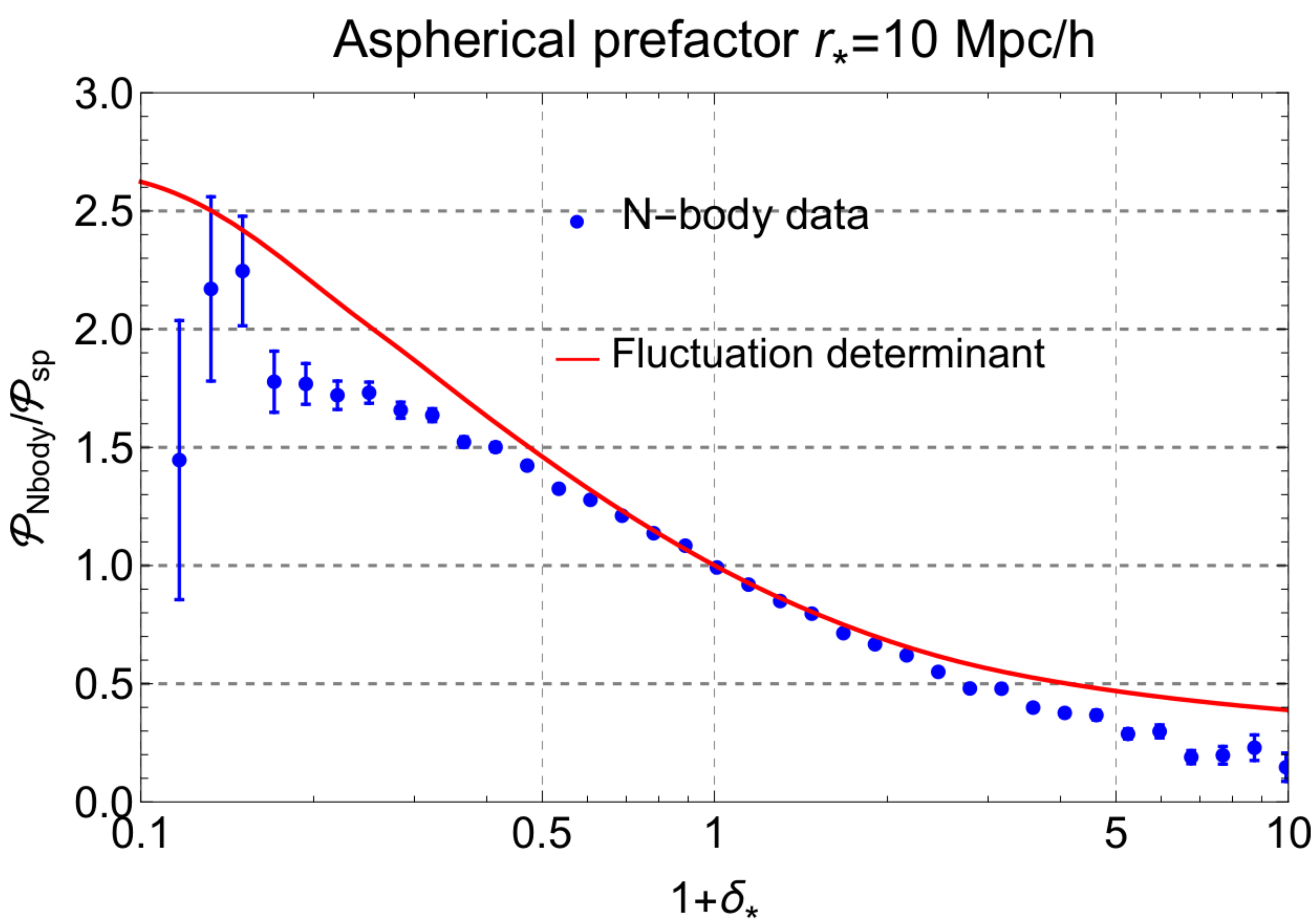
Spherical PDF



- ☑ Unitarity
- ☒ Non-zero mean $\langle\delta\rangle \neq 0$
- ☒ Fail to reproduce data

$$\mathcal{P} = \mathcal{P}_{SP} \times \mathcal{A}_{ASP}$$

Aspherical prefactor (deterministic part)



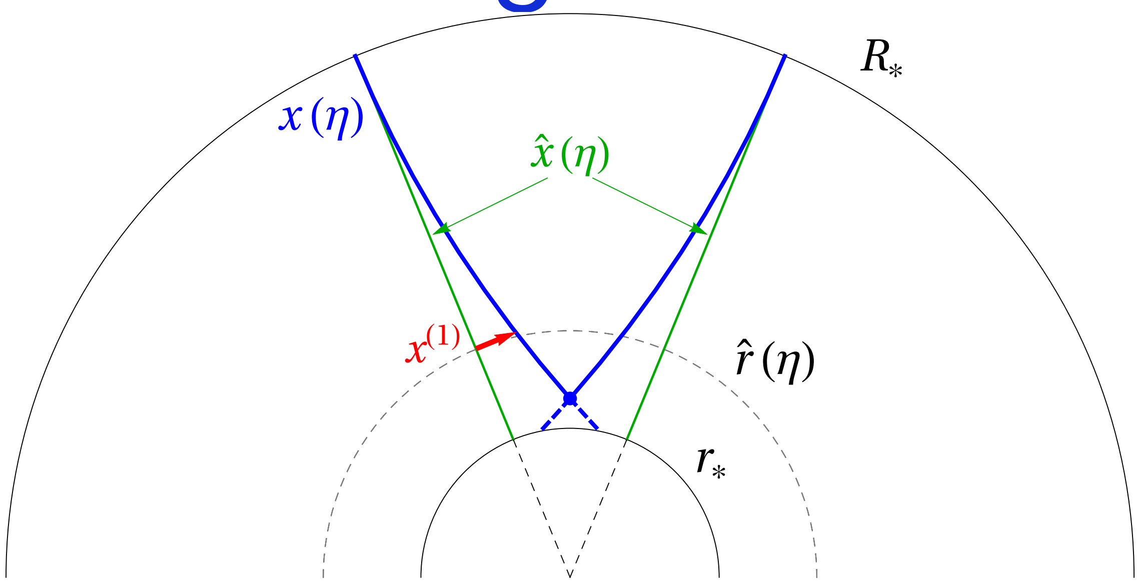
- ☑ Unitarity
- ☑ Zero mean $\langle \delta \rangle = 0$
- ☑ Weakly depend on cosmology
- ☒ Overpredicts the data in tails

$$\mathcal{P} = \mathcal{P}_{SP} \times \mathcal{A}_{ASP}$$

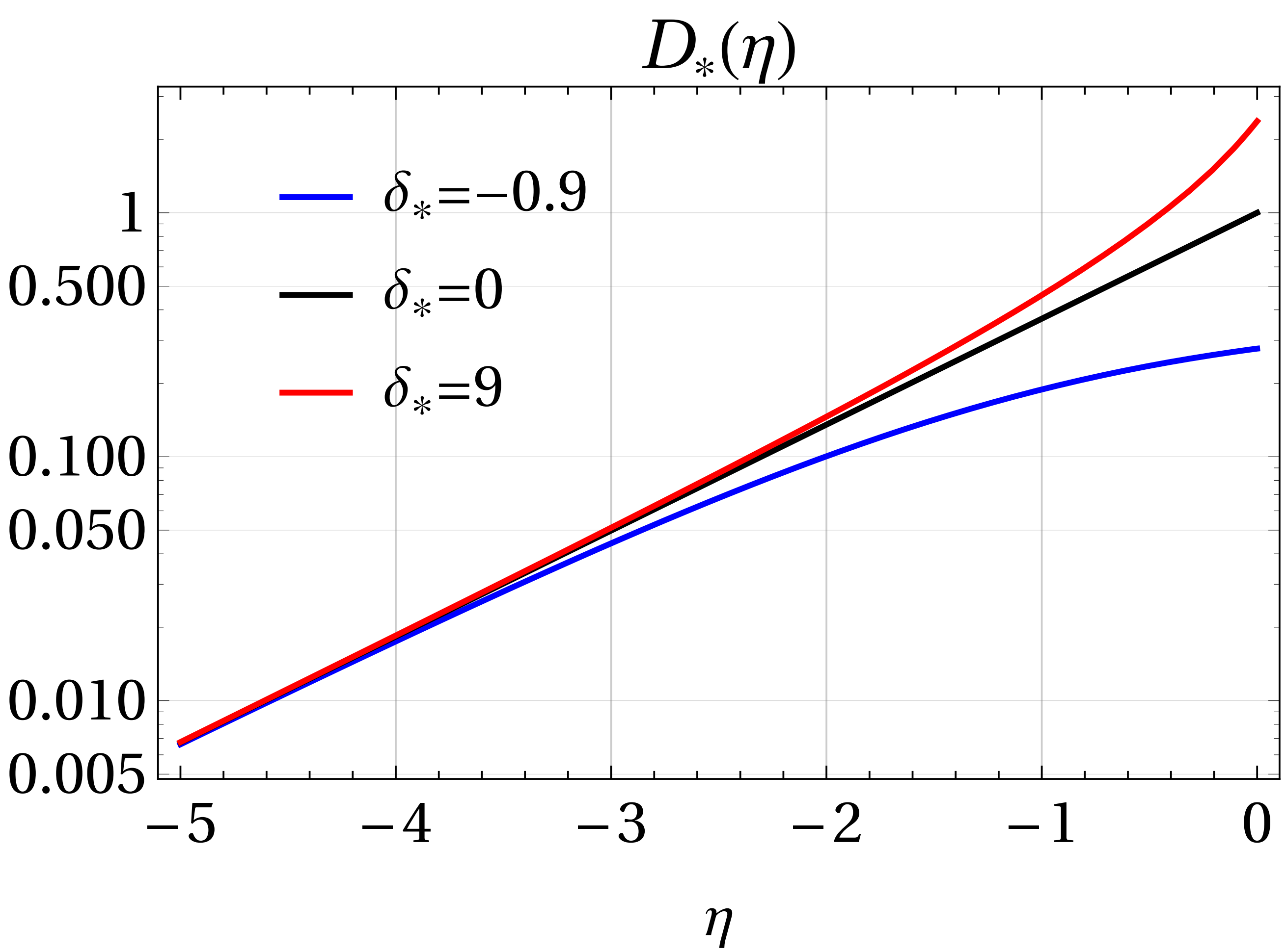
Renormalization of UV physics is required

Growth factor in non-linear background

$$\varpi_i^j \equiv \frac{\partial x_i^{(1)}}{\partial \hat{x}_j}$$



$$\langle \varpi^2 \rangle_{k_{\rm sc}} = 4\pi \int^{k_{\rm sc}} [dk] P(k) [D(\eta; R)]^2 \simeq 1$$



$$D_*(\eta) \equiv D(\eta; R_*)$$

Renormalization of UV physics

$$\frac{\partial \delta}{\partial t} + \partial_i\big((1+\delta)u_i\big) = 0$$

$$\frac{\partial u_i}{\partial t} + \mathcal{H}u_i + (u_j\partial_j)u_i + \partial_i\Phi = -\frac{1}{1+\delta}\partial_j\tau_{ij}$$

$$\delta_{\rm tot} = \delta + \delta^s$$
$$u_{{\rm tot},i} = u_i + u_i^s$$
...

D. Baumann et al., 2012

$$\tau_{ij} = (1+\delta)\sigma^l_{ij} + \frac{2}{3\mathcal{H}^2}\left([\partial_i\Phi^s\partial_j\Phi^s]^l - \frac{1}{2}\delta_{ij}[\partial_k\Phi^s\partial_k\Phi^s]^l\right)$$

'kinetic' part

'potential' part

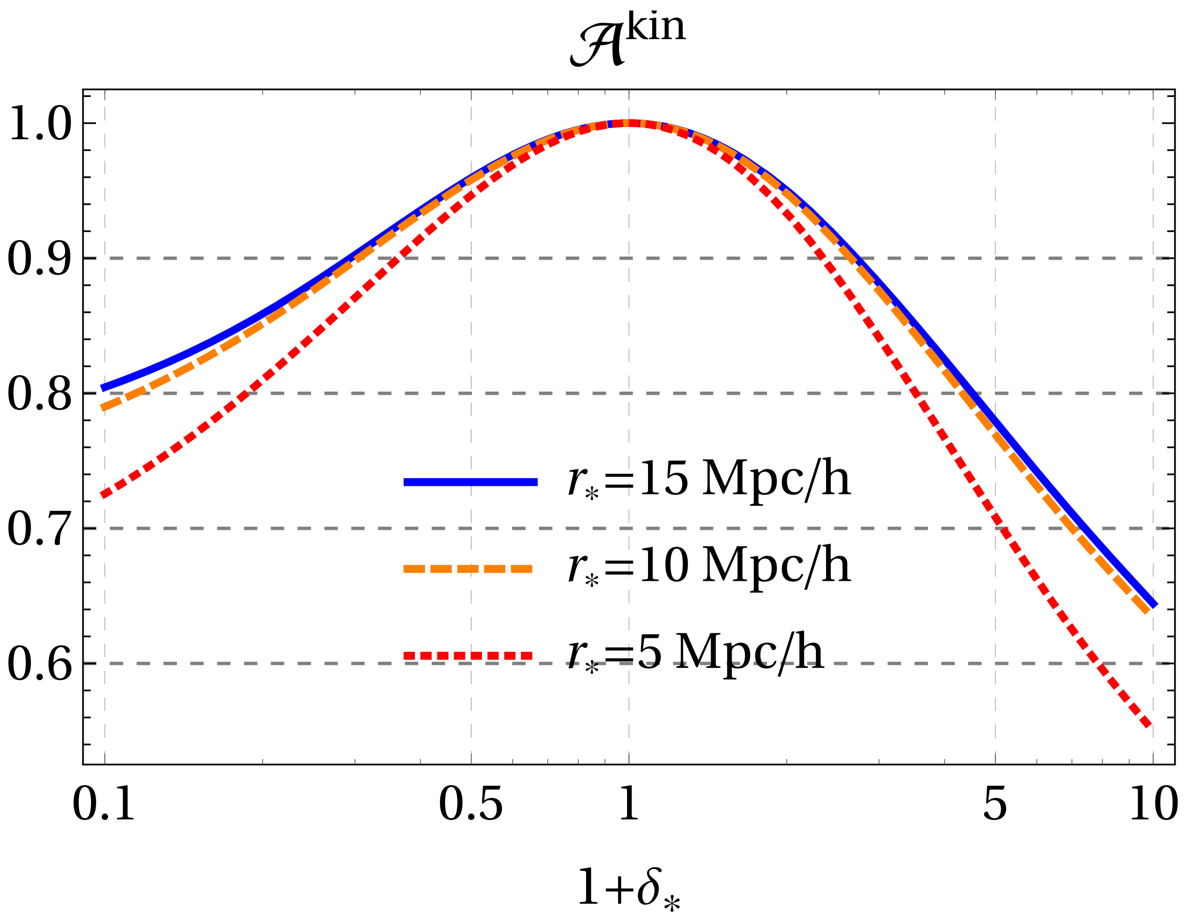
$$\sigma^l_{ij} = \frac{\int (v_i - u^l_i)(v_j - u^l_j) f^l \, d^3p}{\int f^l \, d^3p}\,, \qquad v_i \equiv \frac{p_i}{am}$$

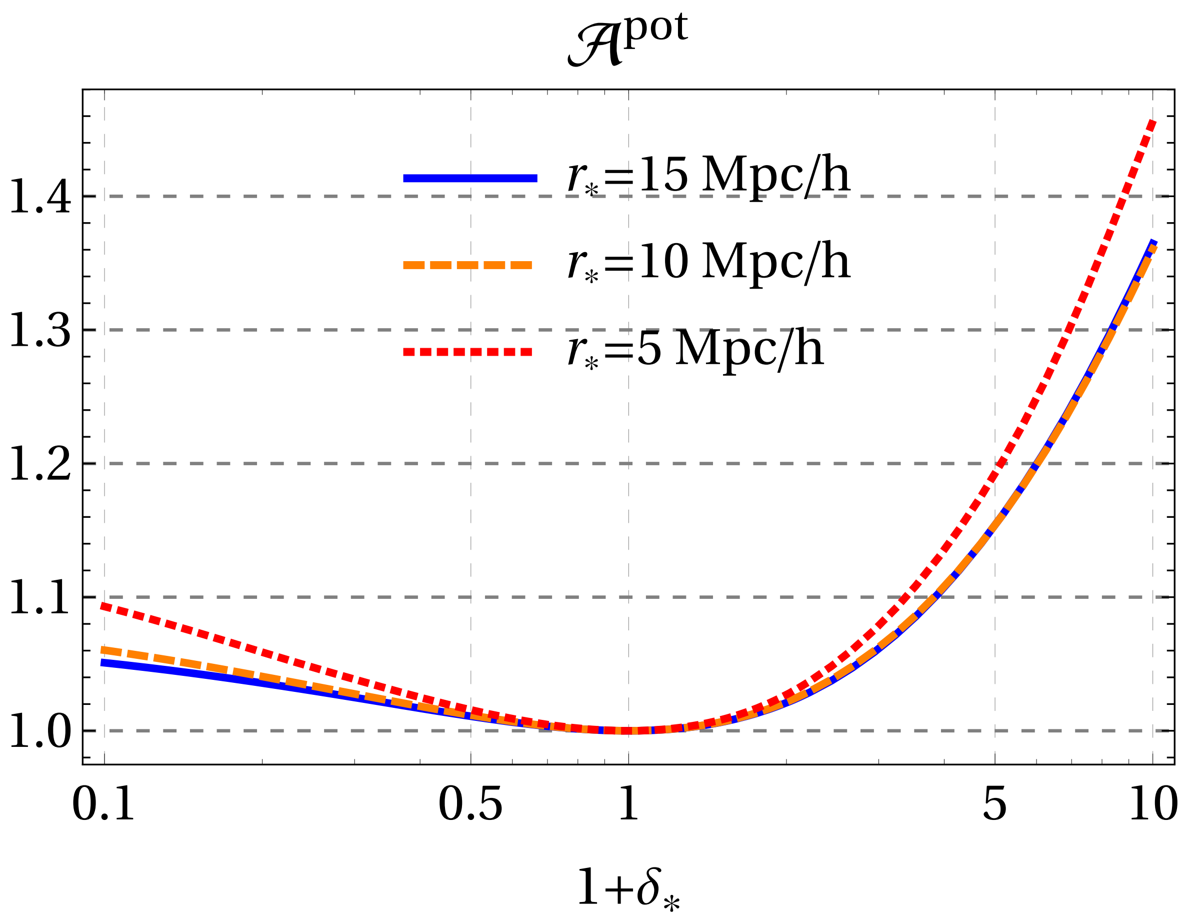
Renormalization

$$\tau^{\rm kin} \rightarrow \zeta^{\rm kin}\,[D(\eta,R)]^{m-2}\,\tau^{\rm kin}\Big|_{\rm fluid}$$
$$\tau^{\rm pot} \rightarrow \zeta^{\rm pot}\,[D(\eta,R)]^{m-2}\,\tau^{\rm pot}\Big|_{\rm fluid}$$

Counterterm refactor

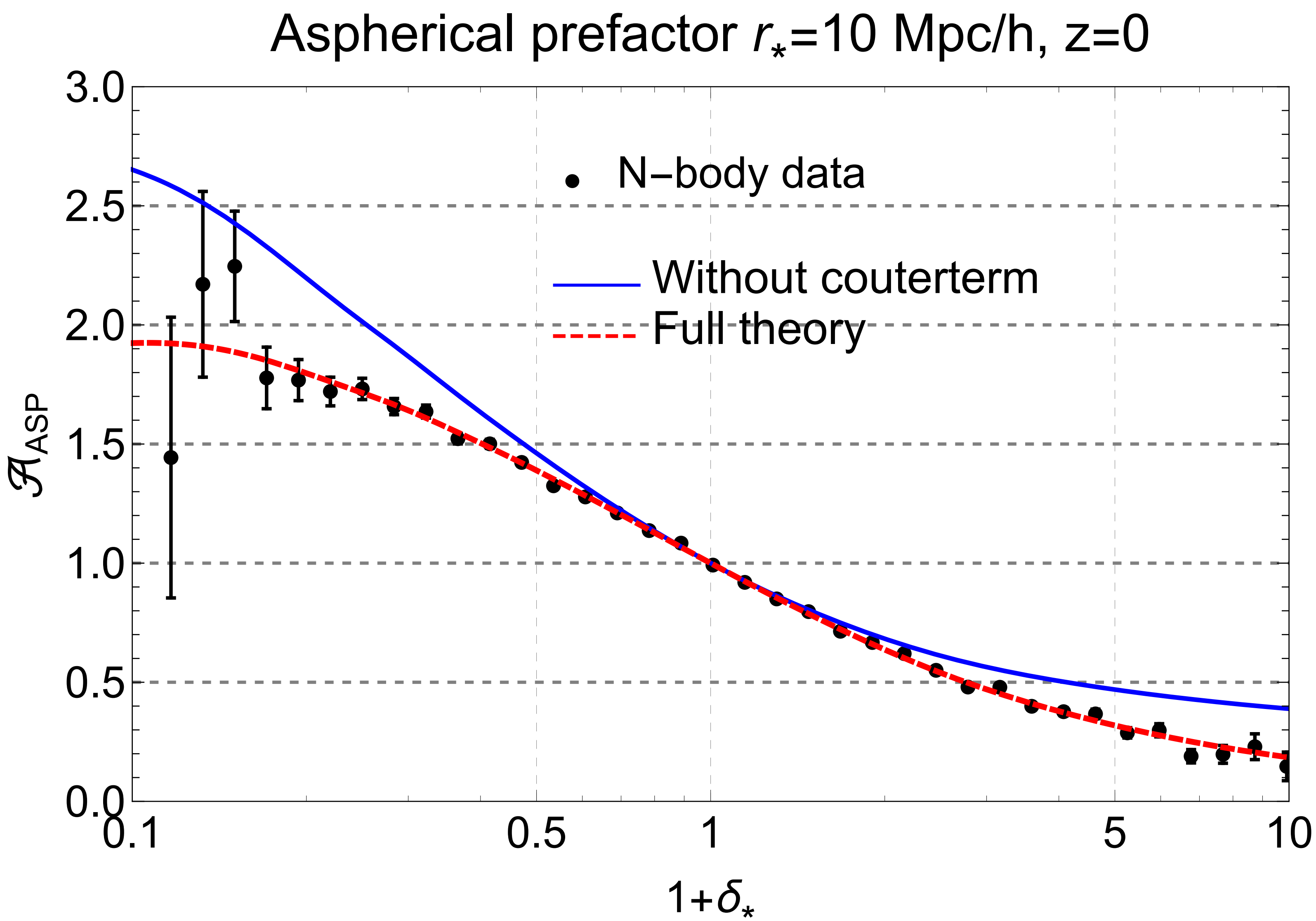
$$\zeta^{\rm kin} = \zeta^{\rm pot} = (1\,{\rm Mpc}/h)^2, \quad m = 2.33$$





Renormalised theory

$$\mathcal{P}(\delta_*) = \mathcal{P}_{\rm SP} \cdot \prod_{\ell>0} \mathcal{A}_\ell(\delta_*) \cdot \mathcal{A}_{\rm ctr}$$



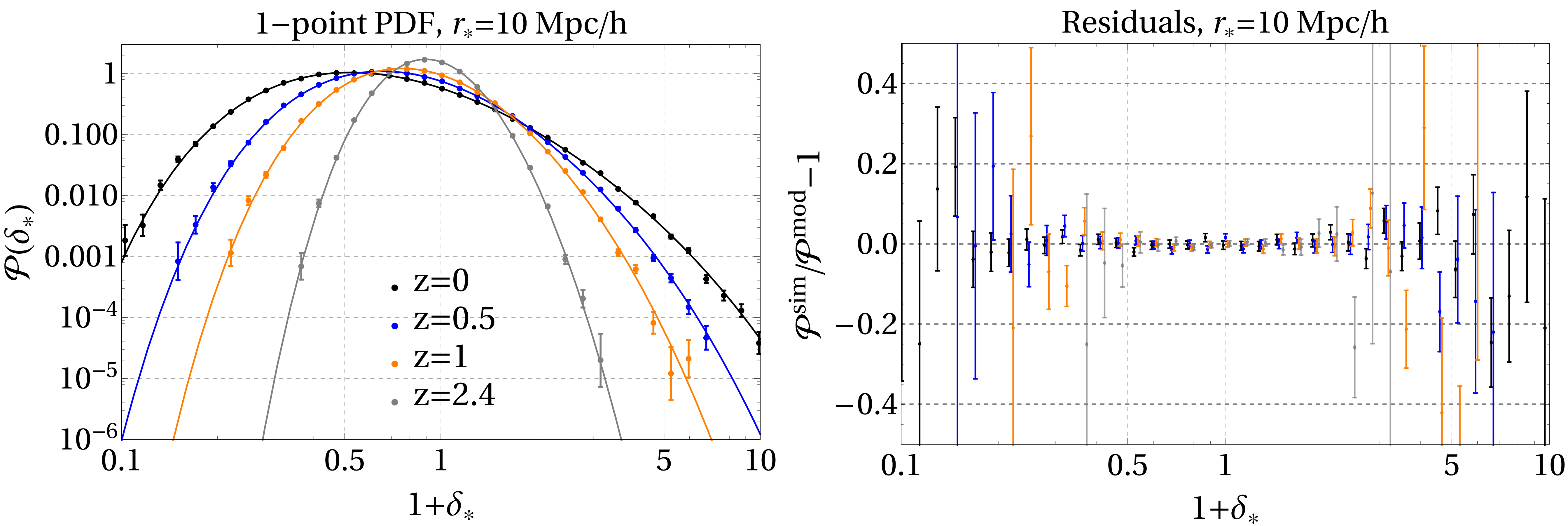
$$\mathcal{A}^{\rm ctr} = \mathcal{A}^{\rm kin} \cdot \mathcal{A}^{\rm pot}$$

Model parameters

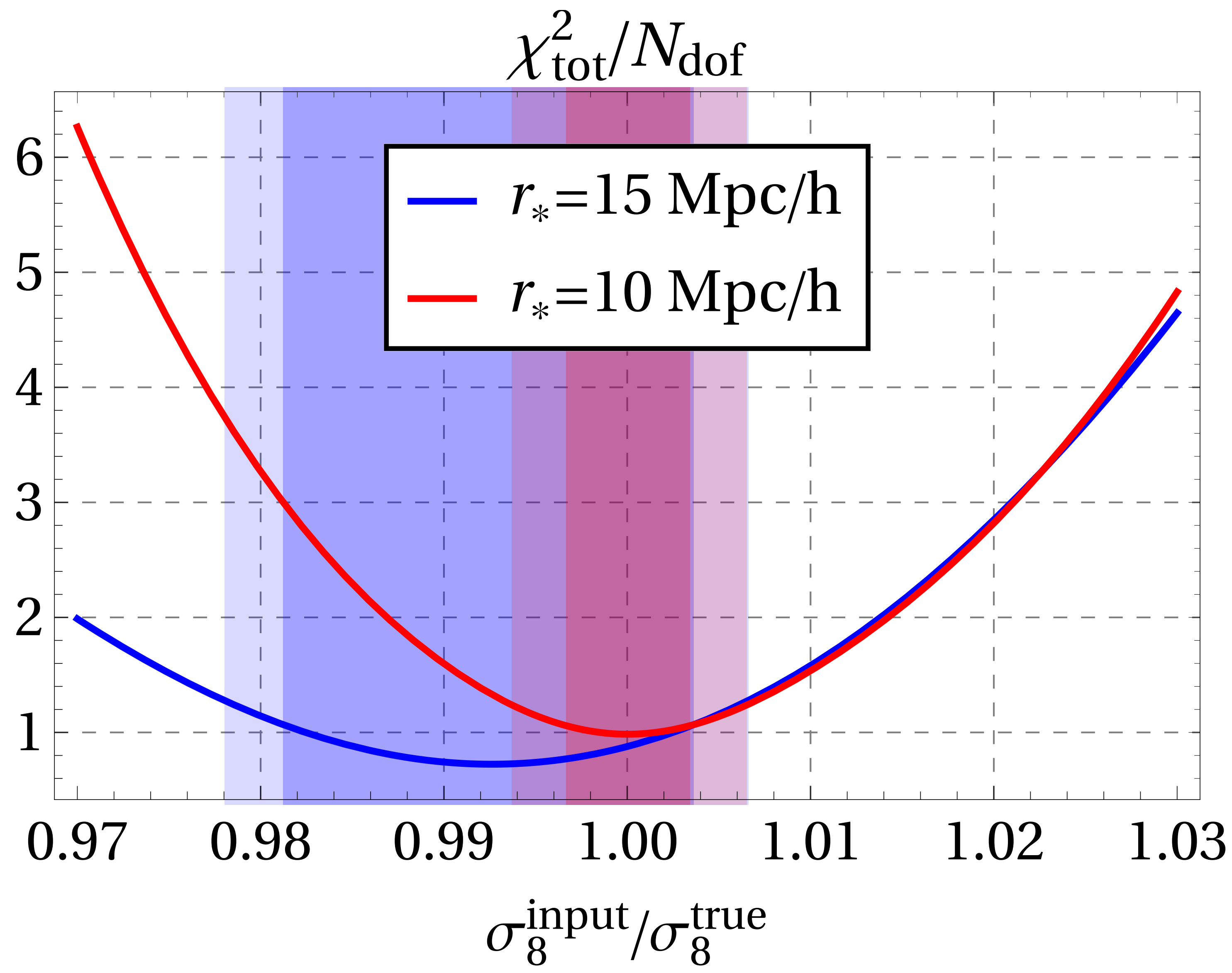
$$\langle \zeta^{\rm kin}, \zeta^{\rm pot}, m \rangle$$

Combined fit

$$\chi^2_{\text{best-fit}}/N_{\text{dof}} = 0.99 \ (0.6\sigma)$$



Sensitivity to σ_8



PDF is sensitive to the value of σ_8 at sub-per cent level

Conclusions

- ☑ Renormalization of UV physics is required to reliable predict the PDF
- ☑ Three-parametric model for couterterm prefactor is in excellent agreement with N-body data for $r_* \geq 10\,\mathrm{Mpc}/h$
- ☑ PDF is highly sensitive to the value of σ_8, perhaps at sub-percent level
- ? Non-linear bias and redshift-space distortions must be taken into account before applying to galaxy surveys

Backup

Filtered n-point correlators

		norm − 1	$\langle\delta_*\rangle$	$\langle\delta_*^2\rangle$	σ^2_{EFT}	$\langle\delta_*^3\rangle/\langle\delta_*^2\rangle^2$
$r_* = 15\,\mathrm{Mpc}/h$	$z = 0$	$-6.2\cdot10^{-3}$	$-7.1\cdot10^{-3}$	0.260	0.262	3.35
	$z = 0.5$	$-2.9\cdot10^{-3}$	$-3.1\cdot10^{-3}$	0.153	0.154	3.30
	$z = 1$	$-1.3\cdot10^{-3}$	$-1.3\cdot10^{-3}$	0.095	0.095	3.27
	$z = 2.4$	$-3.3\cdot10^{-5}$	$-5.6\cdot10^{-5}$	0.035	0.035	3.23
$r_* = 10\,\mathrm{Mpc}/h$	$z = 0$	$-3.7\cdot10^{-4}$	$-2.1\cdot10^{-3}$	0.533	0.532	3.63
	$z = 0.5$	$1.2\cdot10^{-4}$	$-6.7\cdot10^{-4}$	0.306	0.304	3.65
	$z = 1$	$2.8\cdot10^{-4}$	$-2.9\cdot10^{-4}$	0.185	0.185	3.56
	$z = 2.4$	$3.3\cdot10^{-4}$	$6.4\cdot10^{-5}$	0.067	0.067	3.45

PDF reproduces the EFT filtered density variance with sub-per cent accuracy