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WESTERN CAPE

# Probing Anisotropic Expansion With Weak-Lensing

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# Introduction

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# Motivation

- Homogeneity and isotropy on large scales is foundational to modern cosmology
- Some dark energy, modified gravity models lead to large-scale anisotropies
- Fundamentally, this assumption must be tested
- Renewed interest in anisotropic cosmologies (e.g. SN Ia measurements, CMB dipole)  
     $\Rightarrow$  Can use weak lensing to probe anisotropies
- Formalism developed by Pitrou, Pereira, & Uzan to estimate B-mode shear generated by anisotropies  
[\[arXiv:1503.01125\]](#), [\[arXiv:1503.01127\]](#)
- Incorporate non-linear corrections and tomography into results of Pitrou et al.

## Lensing Formalism

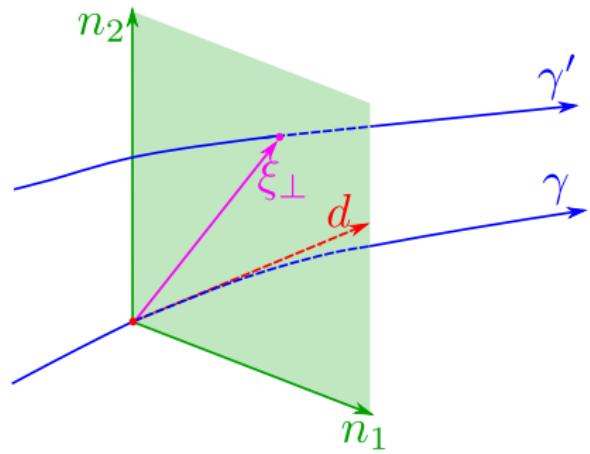
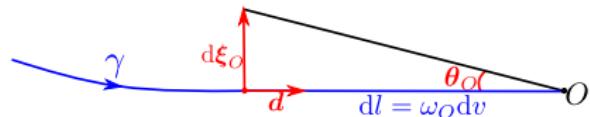
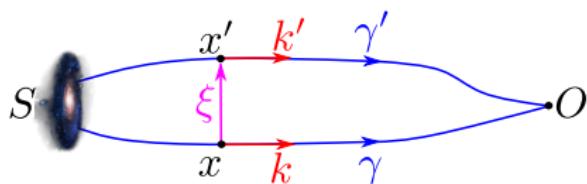
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# Lensing distortions

## Jacobi matrix

- Observed angular size  $\mapsto$  physical separation

$$\xi^A|_S \propto \mathcal{D}^A{}_B \theta^B|_O$$

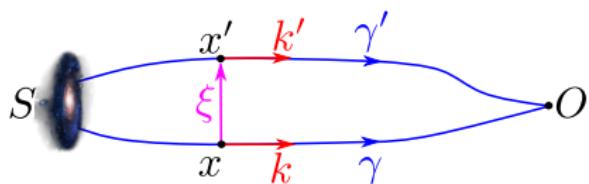


# Lensing distortions

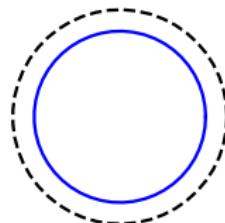
## Jacobi matrix

- Observed angular size  $\mapsto$  physical separation

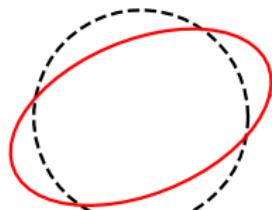
$$\xi^A|_S \propto \mathcal{D}^A{}_B \theta^B|_O$$



Convergence:



Shear:



$$\mathcal{D} \approx \bar{D}_A \left[ \underbrace{\begin{pmatrix} 1 - \kappa & 0 \\ 0 & 1 - \kappa \end{pmatrix}}_{\text{Convergence}} + \underbrace{\begin{pmatrix} 0 & -\psi \\ \psi & 0 \end{pmatrix}}_{\text{Rotation}} + \underbrace{\begin{pmatrix} -\gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{pmatrix}}_{\text{Shear}} \right]$$

- Weak lensing:  $\kappa, \gamma \ll 1$  and  
 $\dot{\psi} = \mathcal{O}(\gamma^2)$   
 $\Rightarrow$  Ignore rotation (usually)

# E-modes, B-modes, and multipoles

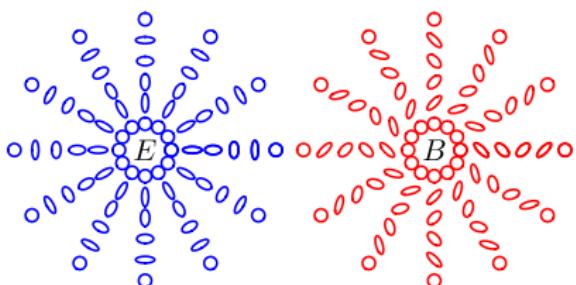
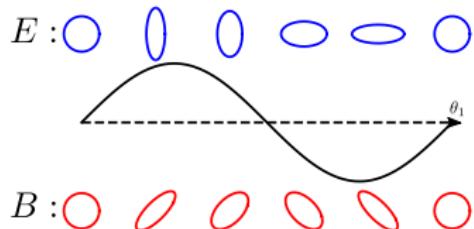
- Expand  $\kappa$  in spherical harmonics

$$\kappa = \sum_{\ell,m} \kappa_{\ell m} Y_{\ell m}$$

- Expand  $\gamma$  in spin-weighted spherical harmonics

$$\gamma^\pm = \gamma_1 \pm i\gamma_2 = \sum_{\ell,m} (E_{\ell m} \pm iB_{\ell m}) Y_{\ell m}^{\pm 2}$$

- $E$  = even parity,  $B$  = odd parity
- No B-modes on large scales (FLRW)\*



# Anisotropic Spacetime

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# Bianchi-I universes

## Metric

$$ds^2 = a^2(-d\eta^2 + \gamma_{ij}dx^i dx^j)$$

- $a$  = scale factor,  $\gamma_{ij}$  = spatial metric
- Hubble rate:  $\mathcal{H} \equiv \frac{a'}{a}$
- Spatial shear:  $\sigma_{ij} \equiv \frac{1}{2}\gamma'_{ij}$

## Evolution

$$\mathcal{H}^2 = \frac{1}{3}\kappa a^2 \rho + \frac{1}{6}\sigma^2$$

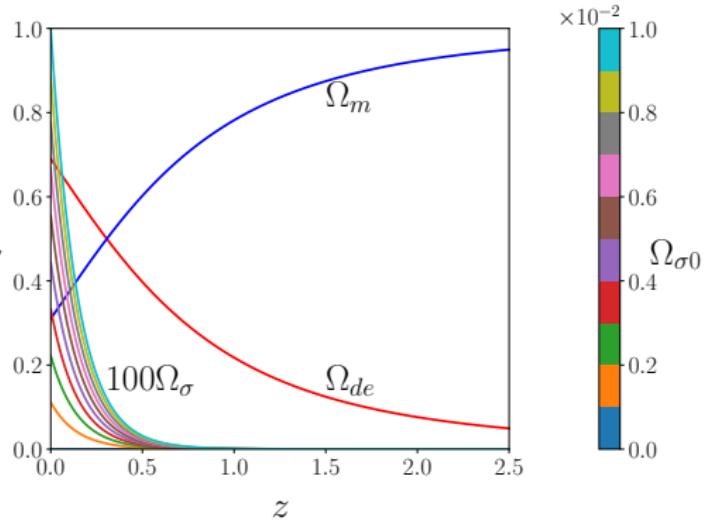
$$(\sigma_j^i)' = -2\mathcal{H}\sigma_j^i + \kappa\Pi_j^i$$

## Dark Energy

$$T_{\mu\nu}^{de} = (\rho_{de} + P_{de})u_\mu u_\nu + P_{de}g_{\mu\nu} + \Pi_{\mu\nu}$$

Anisotropic stress

- EoS:  $P_{de} = -\rho_{de}$
- Anisotropic stress model:  
 $\Pi_j^i \propto \Omega_{de} W_j^i$



# Weak shear limit and perturbation scheme

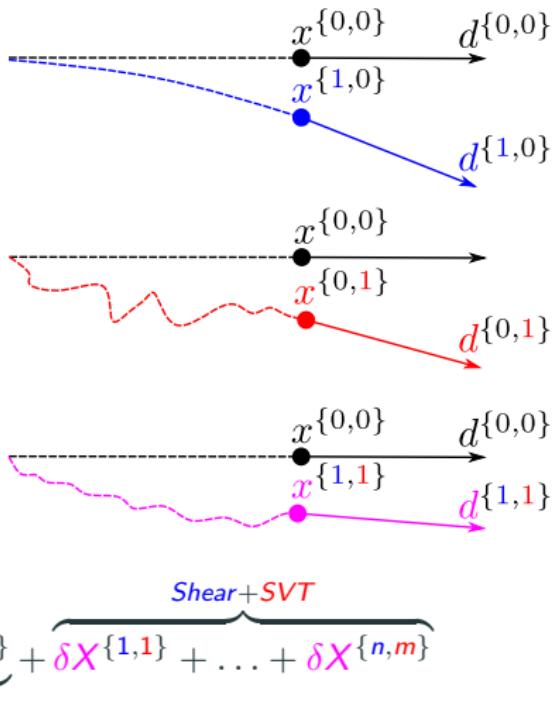
## Perturbed metric

$$ds^2 = a^2 \left[ -(1 + 2\Phi)d\eta^2 + 2\bar{B}_i dx^i d\eta + (\gamma_{ij} + h_{ij})dx^i dx^j \right]$$

## Perturbation scheme

- Treat  $\sigma_{ij}/\mathcal{H} \ll 1$  as perturbation along with scalar-vector-tensor (SVT) perturbations
- SVT:  $\Phi, \bar{B}^i, h_{ij} = -2\Psi(\gamma_{ij} + \frac{\sigma_{ij}}{\mathcal{H}}) + 2E_{ij}$
- Two-fold perturbation  $\{n, m\}$  for shear and SVT\*:

$$\chi^{\{n,m\}} = \underbrace{\chi^{\{0,0\}}}_{\text{FLRW}} + \overbrace{\delta\chi^{\{0,1\}}}^{\text{SVT}} + \underbrace{\delta\chi^{\{1,0\}}}_{\text{Shear}} + \overbrace{\delta\chi^{\{1,1\}} + \dots + \delta\chi^{\{n,m\}}}_{\text{Shear+SVT}}$$

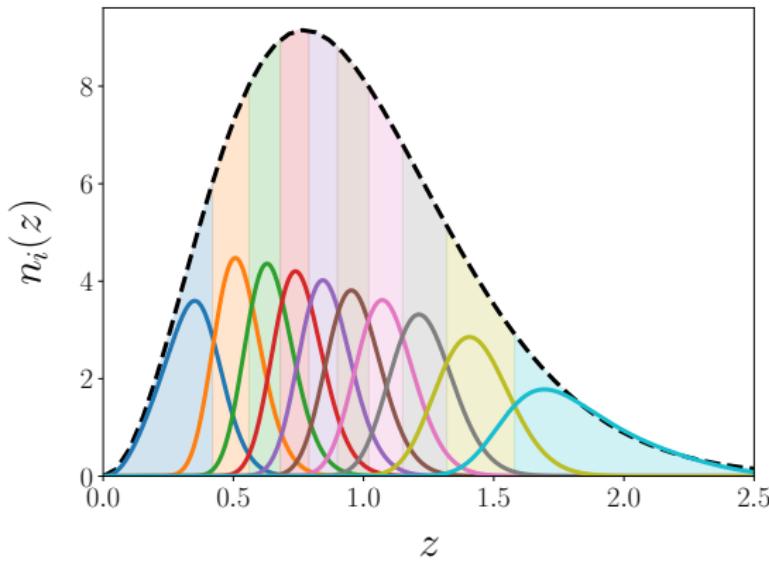


# Results

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# Euclid tomography

- Lensing projects/flattens observables
- Tomography regains some projected info.
- Euclid:
  - 10 equi-populated bins  $10^{-3} \leq z \leq 2.5$
  - Convolve underlying distribution  $n(z)$  with photometric error  $p_{\text{ph}}(z_p|z)$



# Angular power spectra

## Order {1, 0}

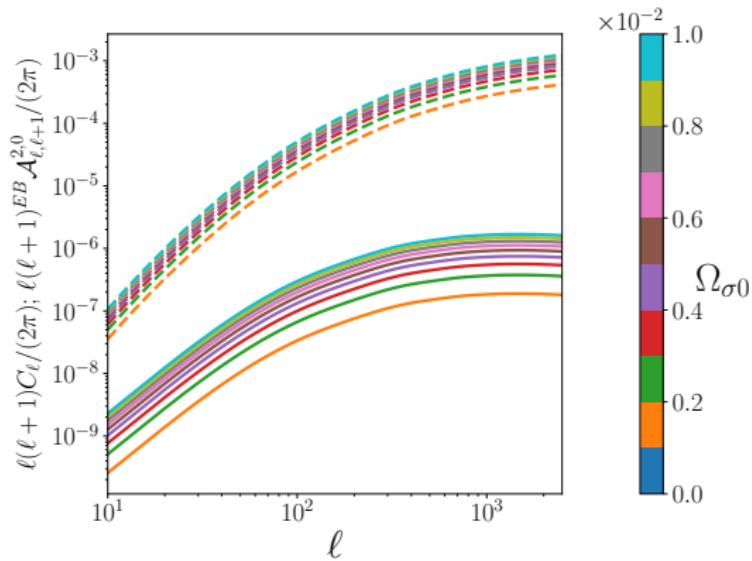
- $\gamma \sim \frac{\sigma}{\mathcal{H}}$
- Fully deterministic  
 $\implies$  no power spectrum

## Order {0, 1}

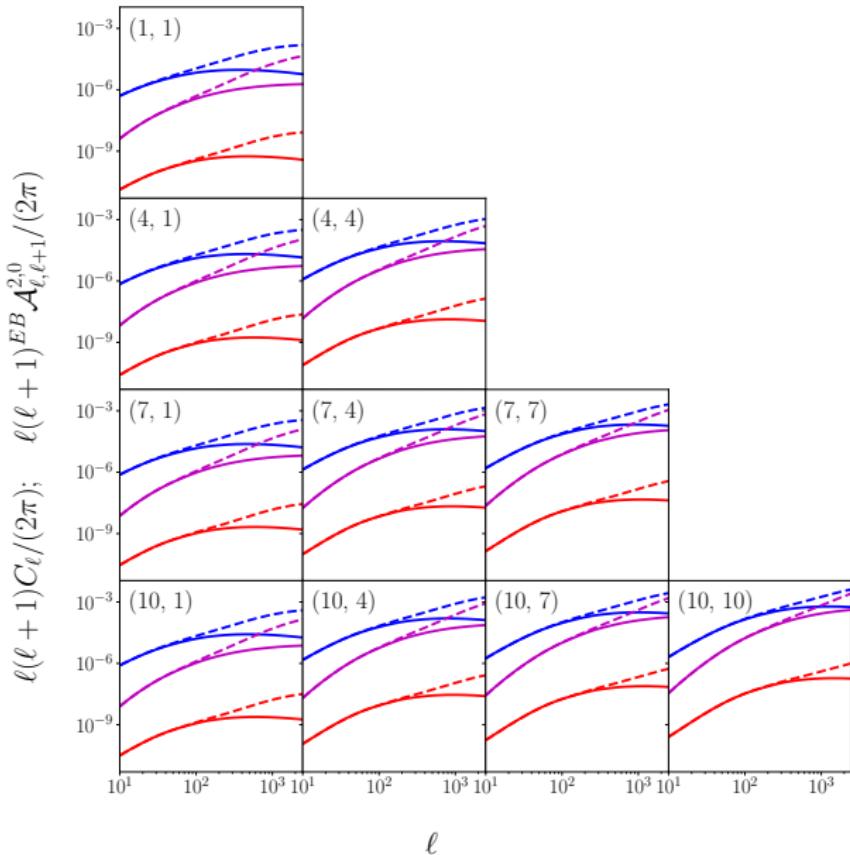
- First order in scalars  
 $\implies$  only E-modes
- Auto:  $C_\ell^{EE} \sim \varphi^2$
- CLASS:  $P(k)$ ,  $T_\varphi(\eta, k)$ ,  
HALOFIT

## Order {1, 1}

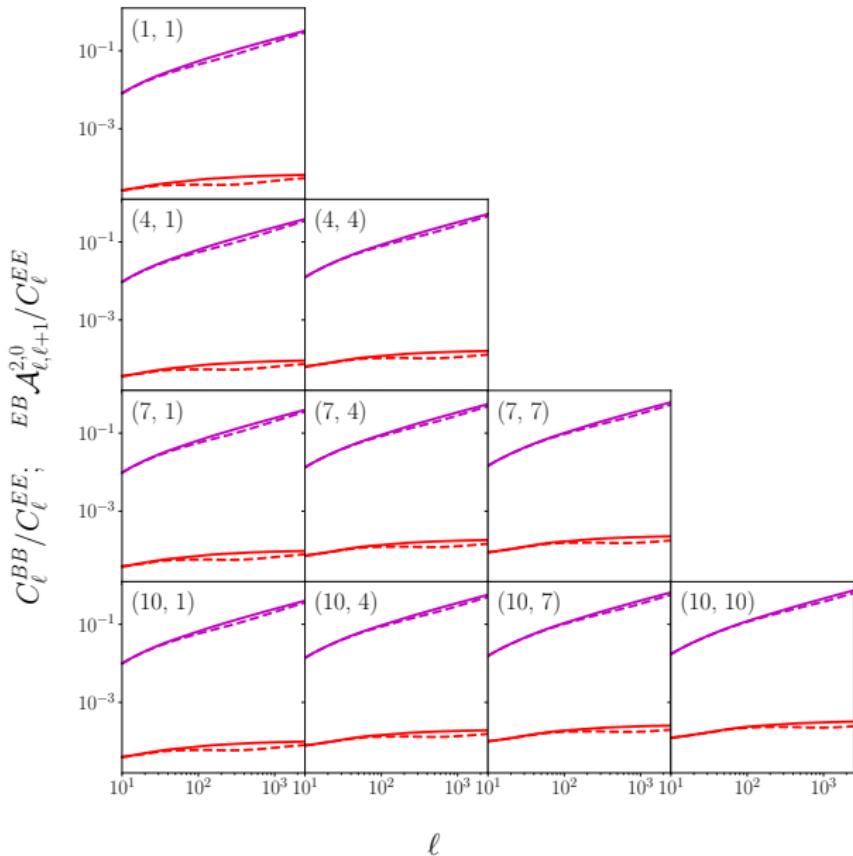
- Post-Born couples  $\sigma$  and scalars  
 $\implies$  Non-zero B-modes!
- Auto:  $C_\ell^{BB} \sim \left(\frac{\sigma}{\mathcal{H}}\right)^2 \varphi^2$
- Cross:  $\langle B_{\ell m} E_{\ell \pm 1 m'}^* \rangle \sim \left(\frac{\sigma}{\mathcal{H}}\right) \varphi^2$   
 $\implies$  off-diagonal (parity)



# Angular power spectra



# Angular power spectra



## Conclusion

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# Concluding remarks

## Summary

- Lightning review of lensing formalism
- Applied perturbation scheme and results Pitrou et al.  
     $\Rightarrow$  incorporated tomography and non-linear corrections
- $B$  auto- and  $E$ - $B$  cross-correlations can be used together in order to constrain late-time anisotropic expansion
- Should construct appropriate estimators for cross-correlations

## Outlook

- Investigate other  $B$ -mode sources (IA, clustering, GW,...)
- Beyond Limber? Forecasting?
- Weak lensing is of immense importance to upcoming surveys
- Hope to place constraints on  $\sigma/\mathcal{H}$  at the percent level

**Questions?**