The peculiar velocity bispectrum

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Sezione di Torino

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Peculiar velocities

- Galaxies' peculiar velocities can be estimated by combining measurements of their observed redshift with a distance indicator that enables us to infer the cosmological redshift independently

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Peculiar velocities

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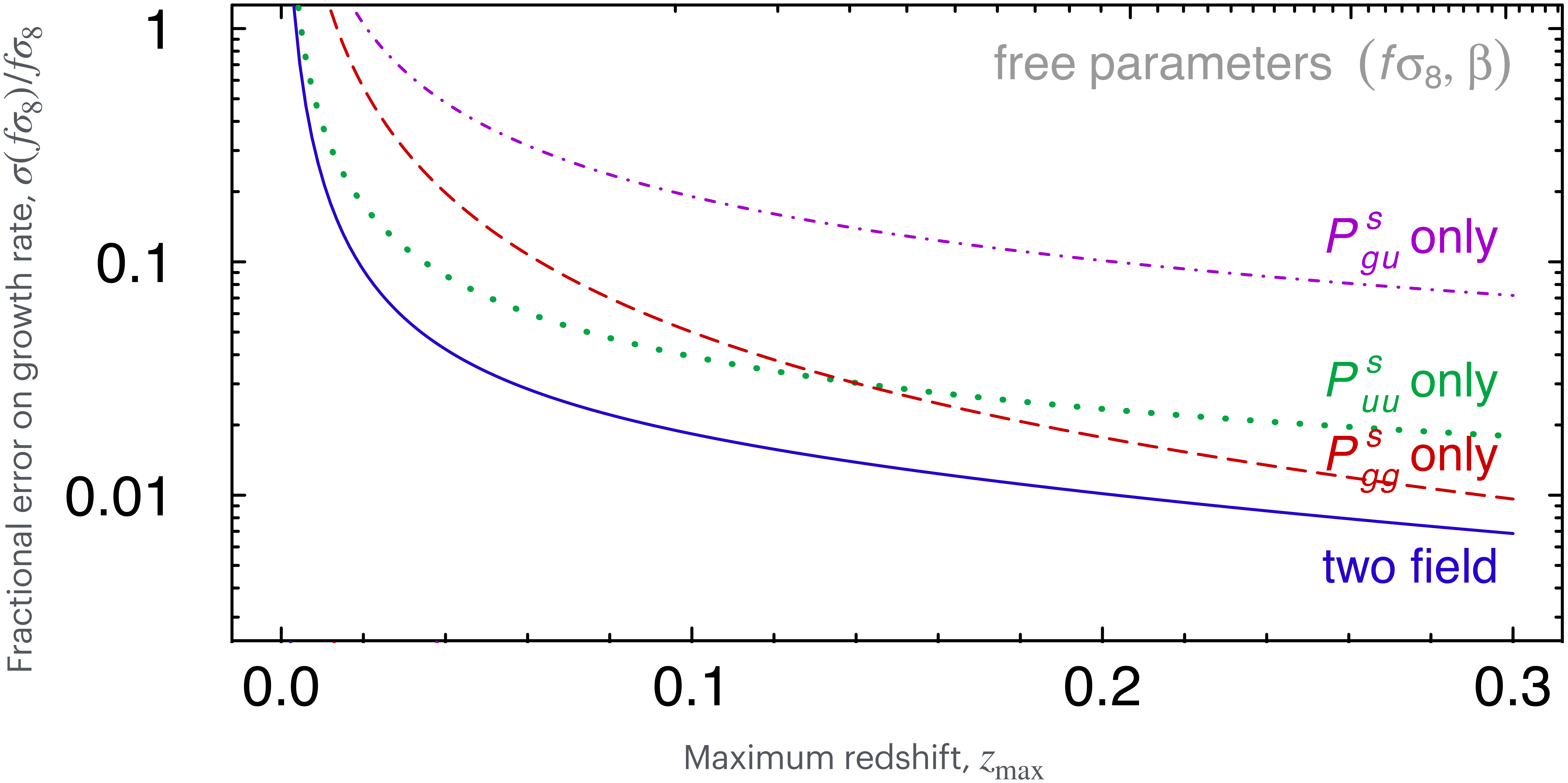

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[Carrick et al. 2015; Qin et al. 2019; Adams & Blake 2020; Said et al. 2020; Lai et al. 2022]

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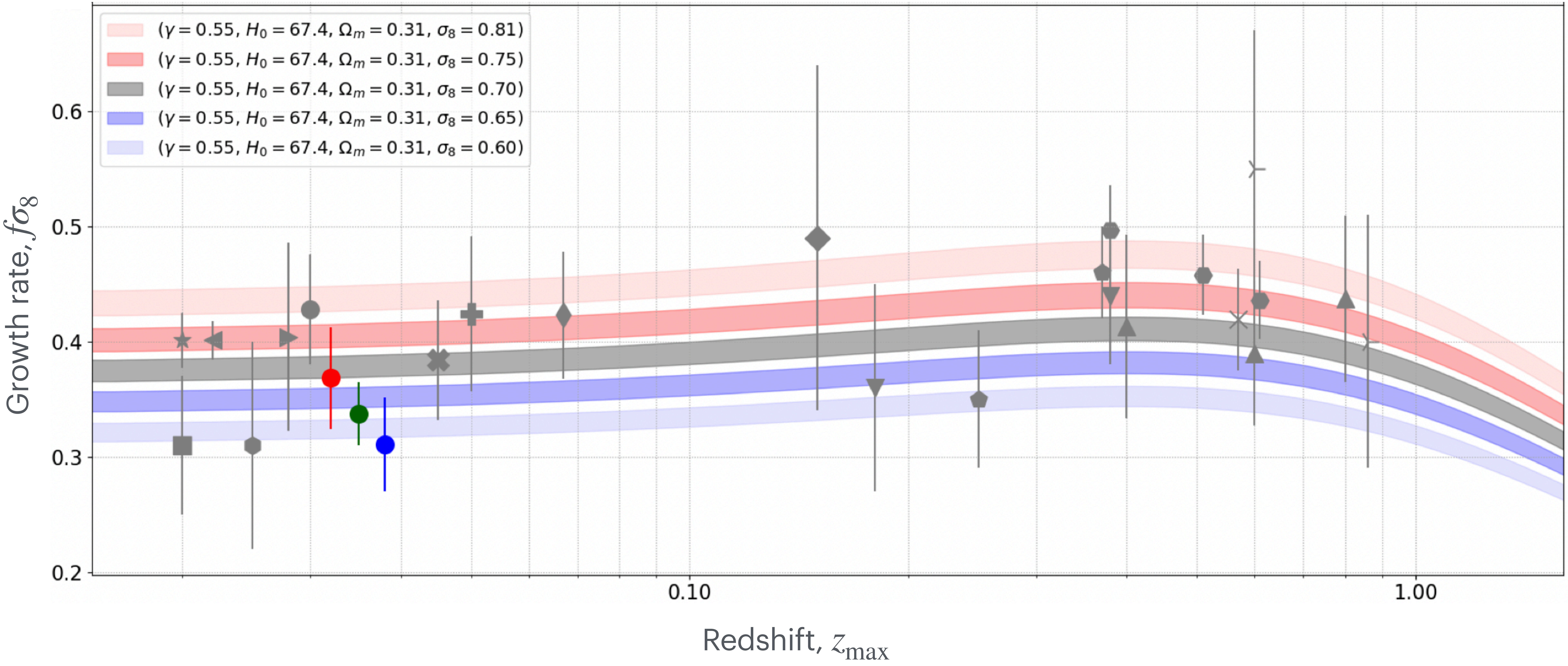

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- What about the bispectrum, then?

Polyspectra formalism

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- Small perturbations can be treated perturbatively

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$$\delta(\boldsymbol{k}, z) = \sum_{m=1}^{\infty} D^m(z)\, \delta^{(m)}(\boldsymbol{k})$$

$$\theta(\boldsymbol{k}, z) = -\mathcal{H}(z)\, f(z) \sum_{m=1}^{\infty} D^m(z)\, \theta^{(m)}(\boldsymbol{k})$$

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Standard perturbation theory primer

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- From the continuity and Euler's equations in Fourier space

$$\begin{bmatrix} \delta^{(m)}(\boldsymbol{k}) \\ \theta^{(m)}(\boldsymbol{k}) \end{bmatrix} = \int_{\boldsymbol{q}_1} \dots \int_{\boldsymbol{q}_m} \delta^{(1)}(\boldsymbol{q}_1) \, \dots \delta^{(1)}(\boldsymbol{q}_m) \begin{bmatrix} F_m(\boldsymbol{q}_1, \dots, \boldsymbol{q}_m) \\ G_m(\boldsymbol{q}_1, \dots, \boldsymbol{q}_m) \end{bmatrix} (2\,\pi)^3 \, \delta_{\mathrm{D}}(\boldsymbol{q}_{1\dots m} - \boldsymbol{k})$$

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- Hence the tree-level power spectrum and bispectrum

$$\langle X(\boldsymbol{k}_1)\, X(\boldsymbol{k}_2)\rangle = \langle X^{(1)}(\boldsymbol{k}_1)\, X^{(1)}(\boldsymbol{k}_2)\rangle$$

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Galaxy clustering

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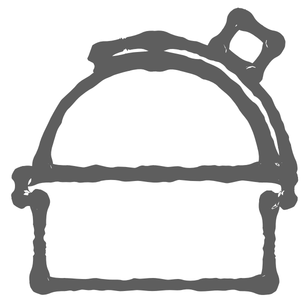

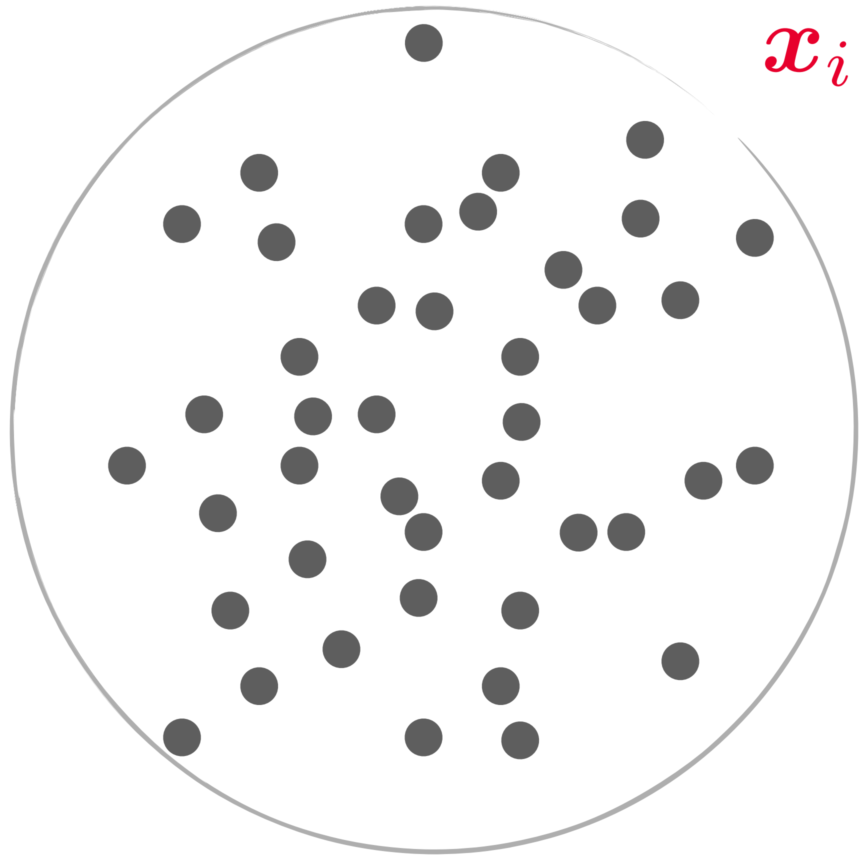

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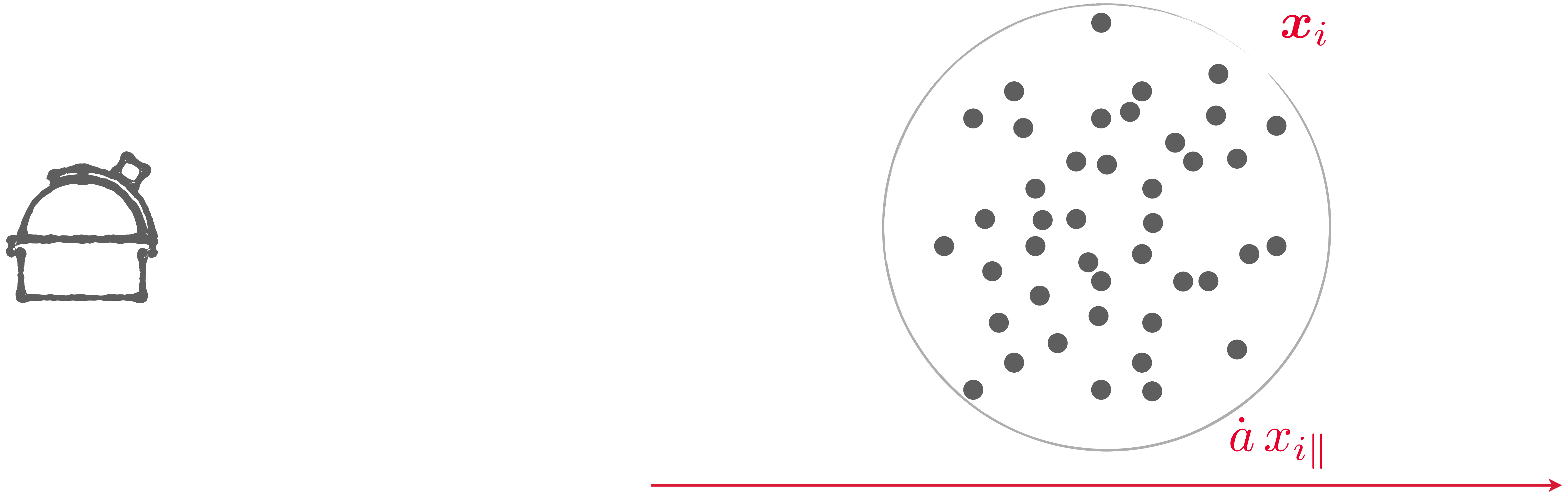

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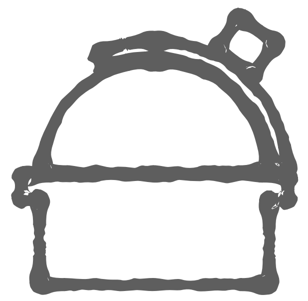

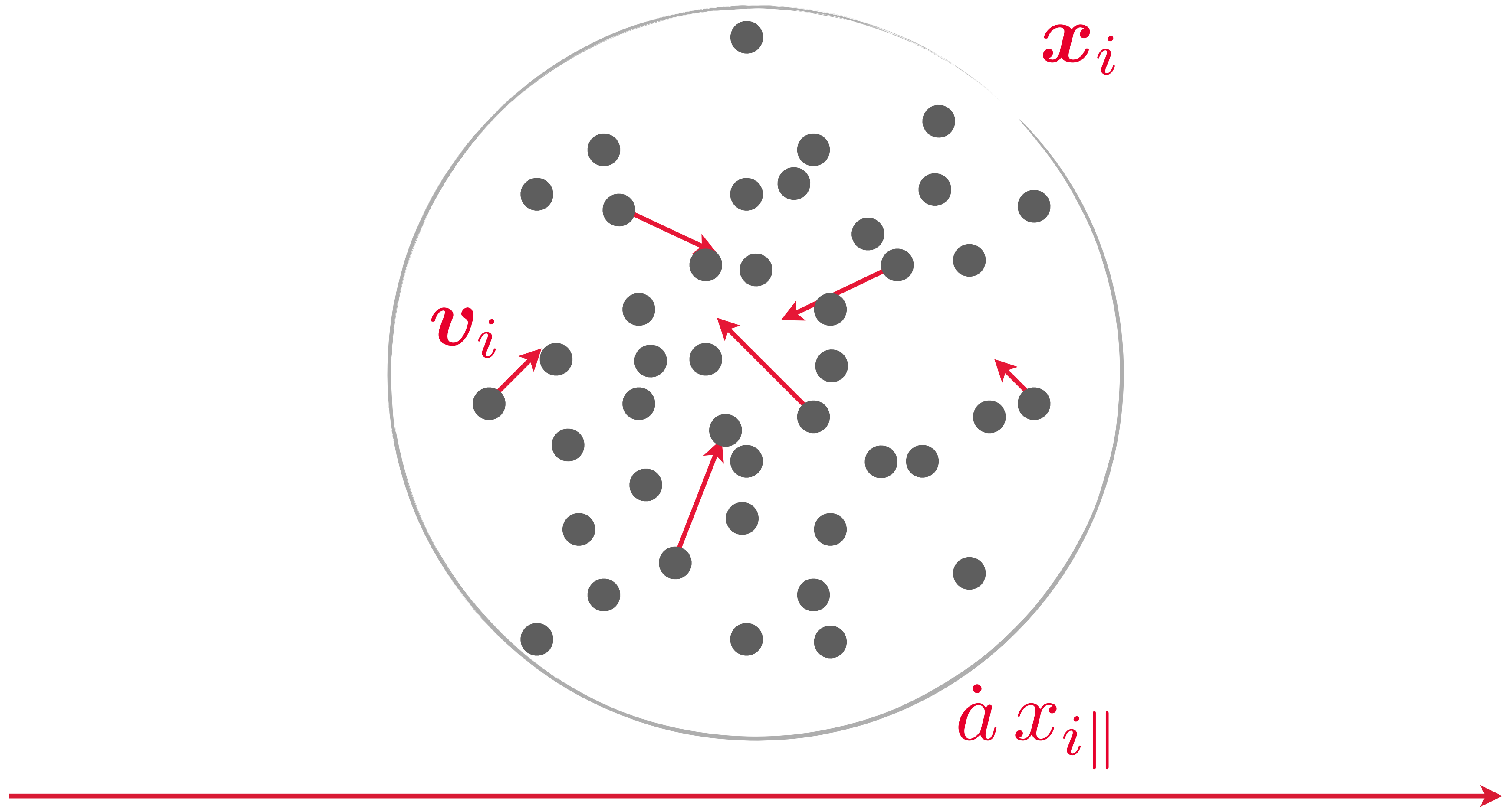

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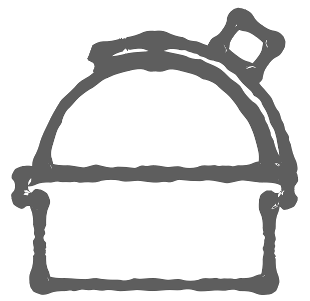

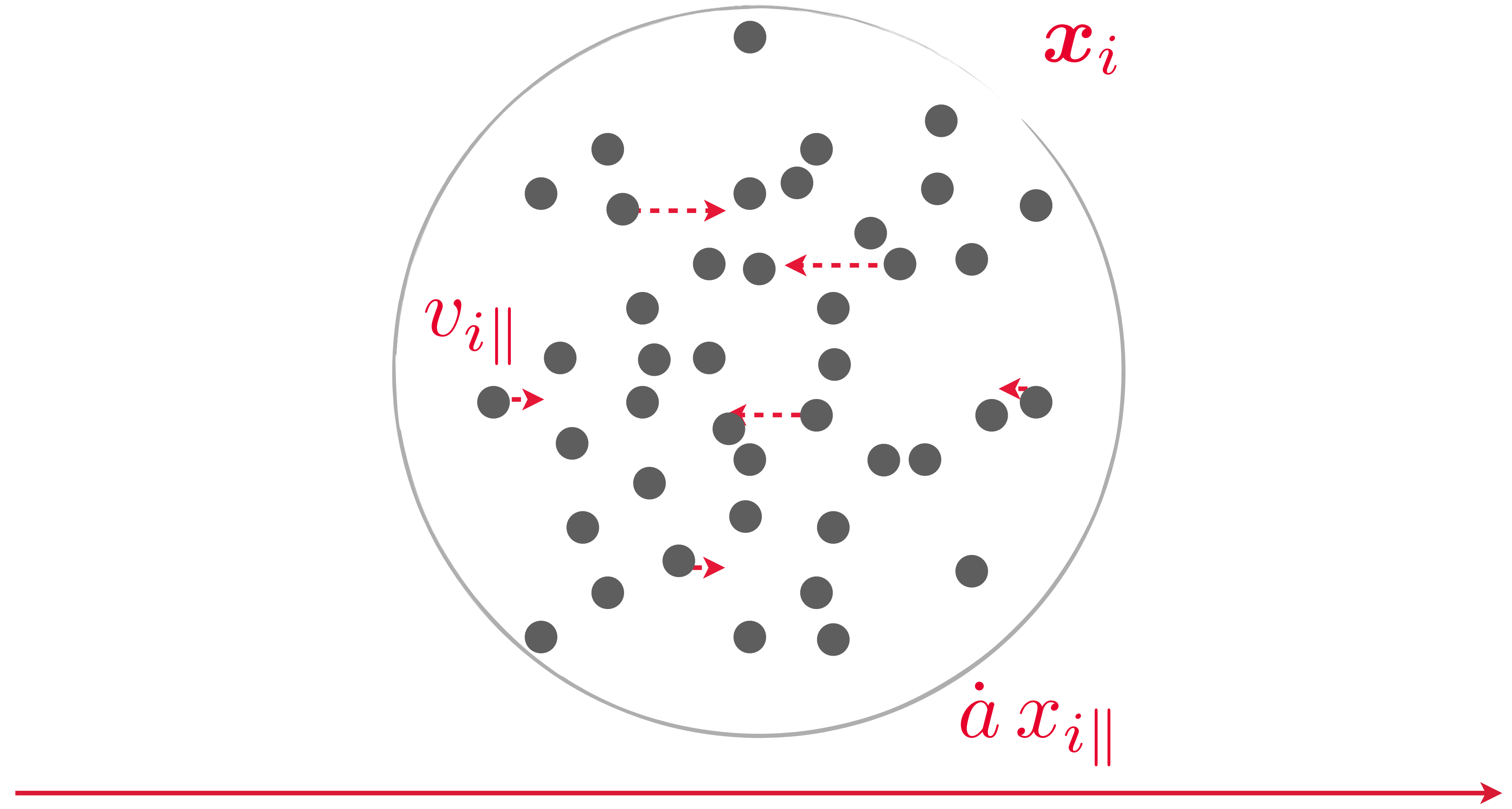

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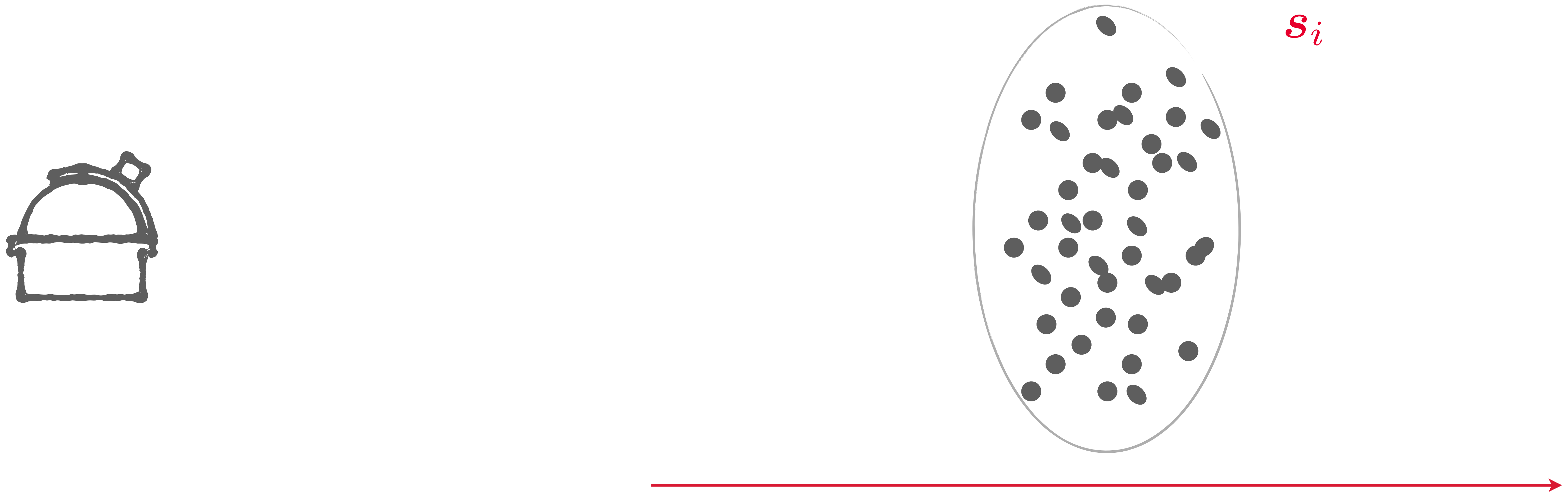

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Galaxy clustering

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- Knowing the real-to-redshift space mapping, we construct redshift-space kernels...

$$\Delta^{(m)}(\boldsymbol{k}) = \int_{\boldsymbol{q}_1} \dots \int_{\boldsymbol{q}_m} \delta^{(1)}(\boldsymbol{q}_1) \dots \delta^{(1)}(\boldsymbol{q}_m)\, \mathcal{Z}^{(m)}(\boldsymbol{q}_1, \dots, \boldsymbol{q}_m)\,(2\,\pi)^3\,\delta_{\rm D}(\boldsymbol{q}_{1\dots m} - \boldsymbol{k})$$

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Galaxy clustering

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- Galaxy clustering kernels in redshift space

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- Analogously to what done before, I computed peculiar-velocity kernels...

$$u^{(m)}(\boldsymbol{k}) = \int_{\boldsymbol{q}_1} \dots \int_{\boldsymbol{q}_m} \delta^{(1)}(\boldsymbol{q}_1) \dots \delta^{(1)}(\boldsymbol{q}_m) \, \mathcal{U}^{(m)}(\boldsymbol{q}_1, \dots, \boldsymbol{q}_m) \, (2\,\pi)^3 \, \delta_{\rm D}(\boldsymbol{q}_{1\dots m} - \boldsymbol{k})$$

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$$\mathcal{U}^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \mathcal{U}^{(1)}(\boldsymbol{k}_{12})\,D\,\left[G_2(\boldsymbol{k}_1,\boldsymbol{k}_2) - \frac{3}{2}\,f\,\mu_1\,\mu_2\,\frac{(k_{12})^2}{k_1\,k_2}\right]$$

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Momentum density

- Momentum density is the density-weighted peculiar velocity field

$$p \coloneqq (1+\varDelta)\,u$$

- ...from which

$$\mathcal{P}^{(1)}(\boldsymbol{k}_1) = \mathcal{U}^{(1)}(\boldsymbol{k}_1)$$

$$\mathcal{P}^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \mathcal{U}^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) + \mathcal{U}^{(1)}(\boldsymbol{k}_{12})\,D\left[\frac{f}{2}\,\mu_1\,\mu_2\,\frac{(k_{12})^2}{k_1\,k_2} + \frac{b_1}{2}\left(\frac{\mu_1}{k_1}+\frac{\mu_2}{k_2}\right)\frac{k_{12}}{\mu_{12}}\right]$$

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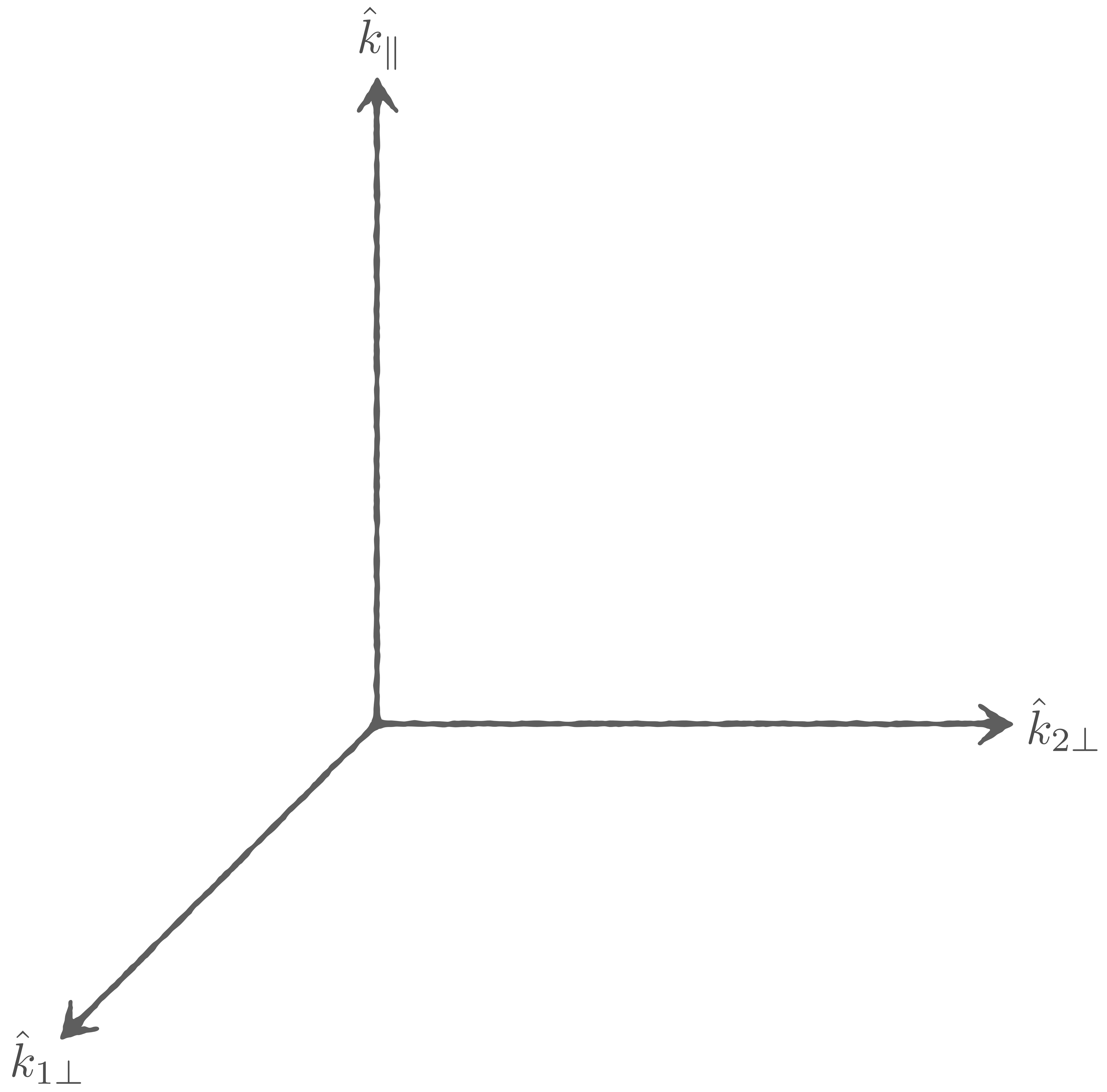
Constructing bispectra



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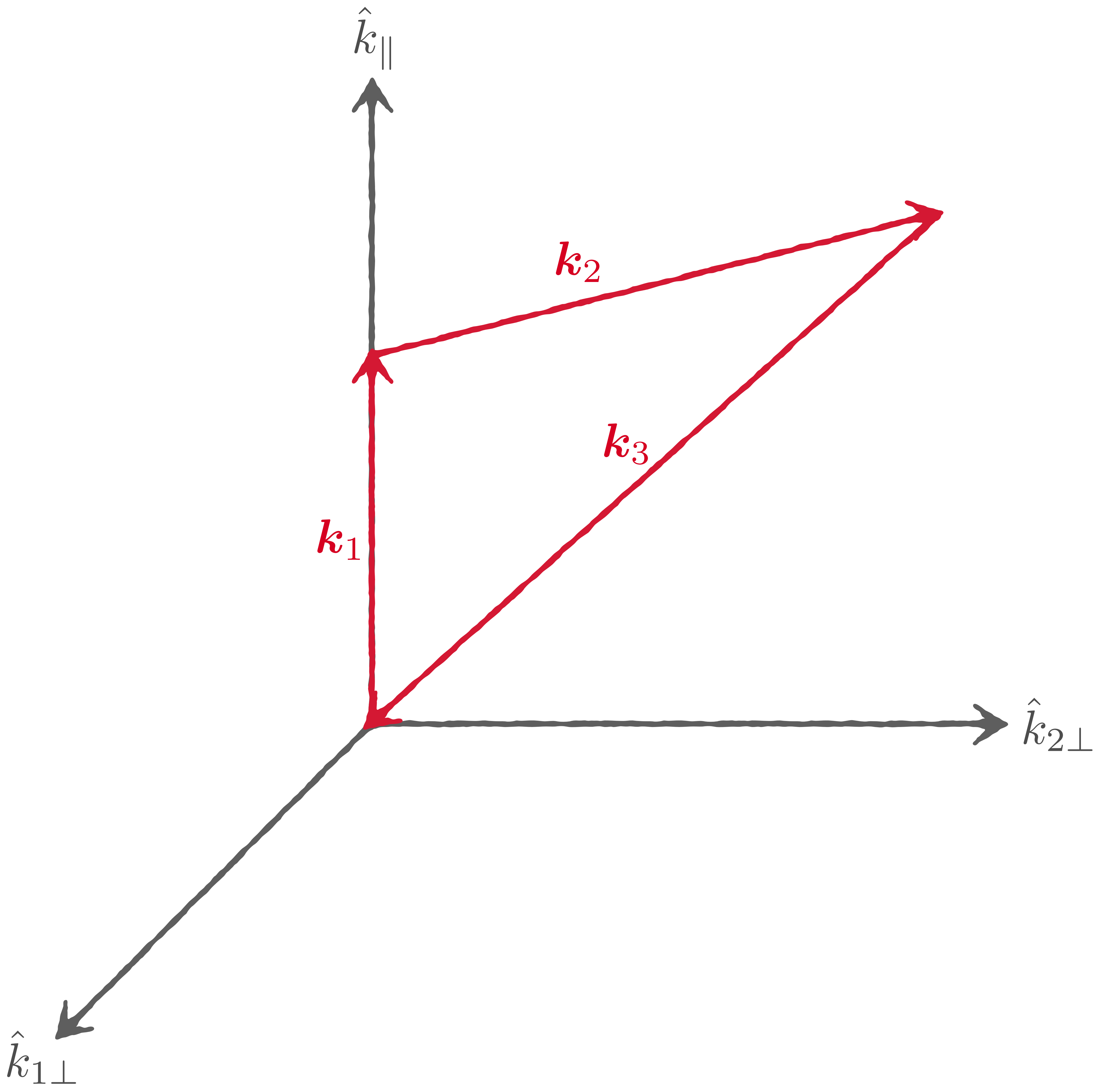

Constructing bispectra

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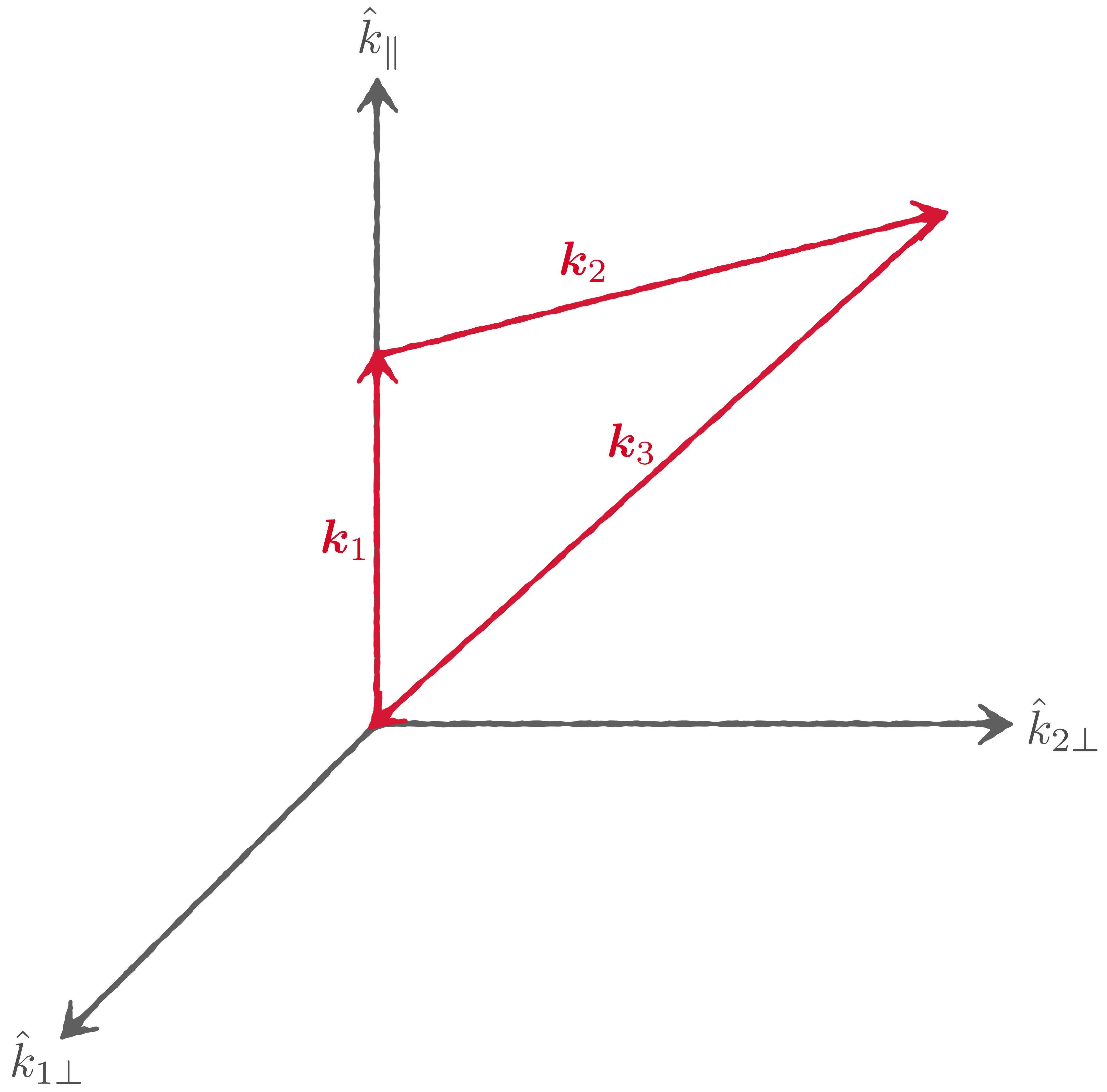
  - $3\ 3\text{D}\ \boldsymbol{k}_i = 9$



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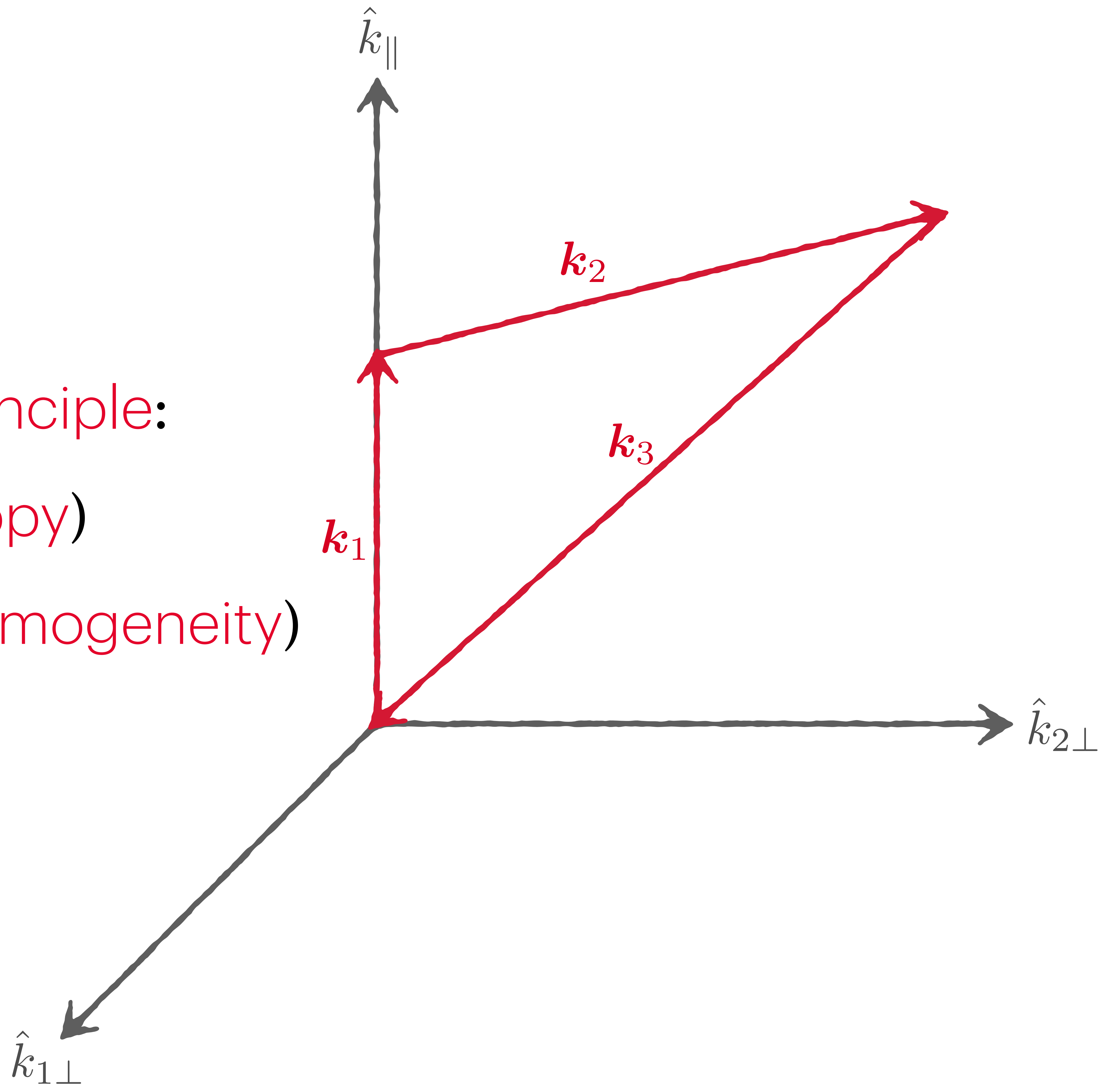

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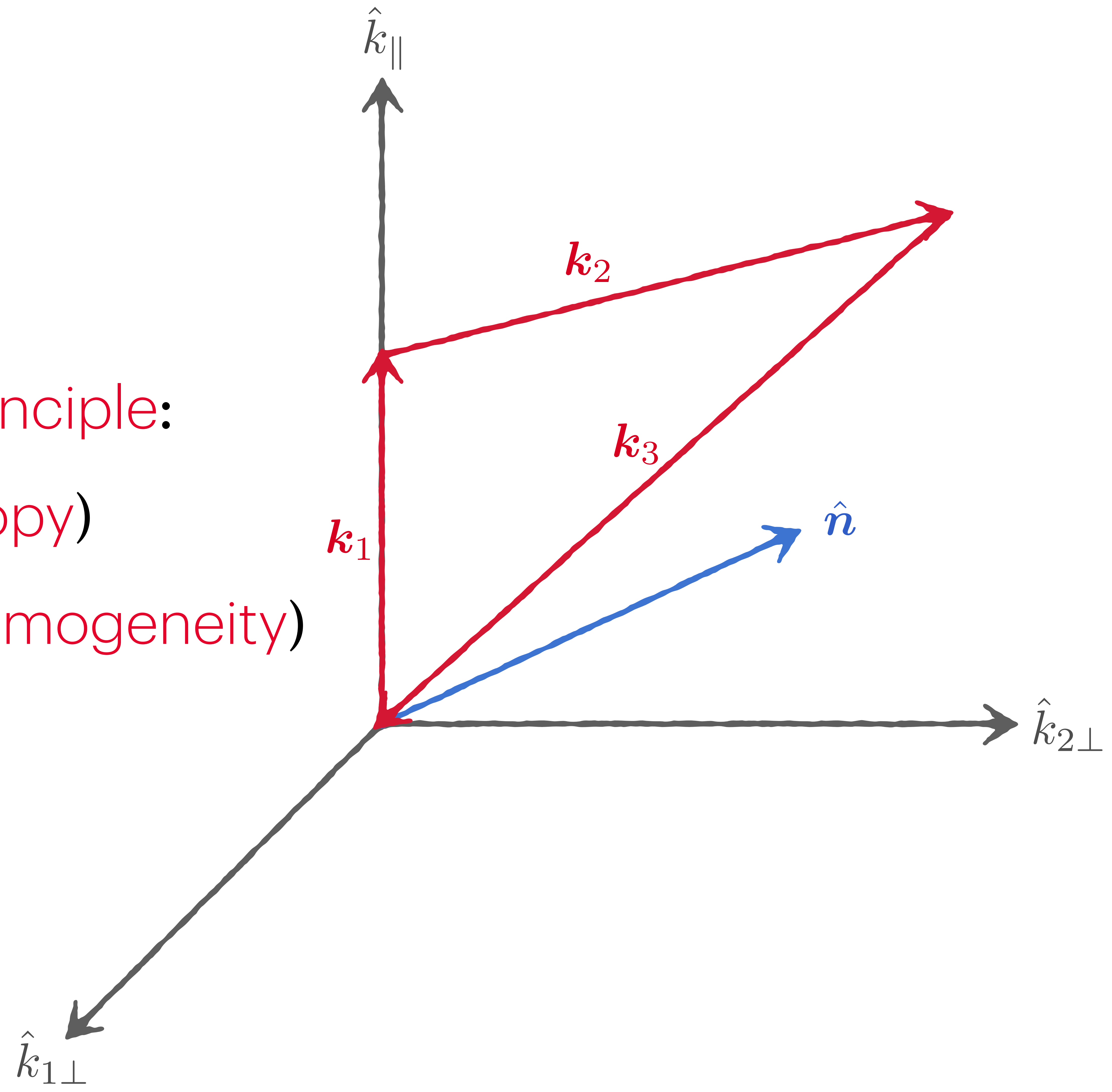

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Constructing bispectra

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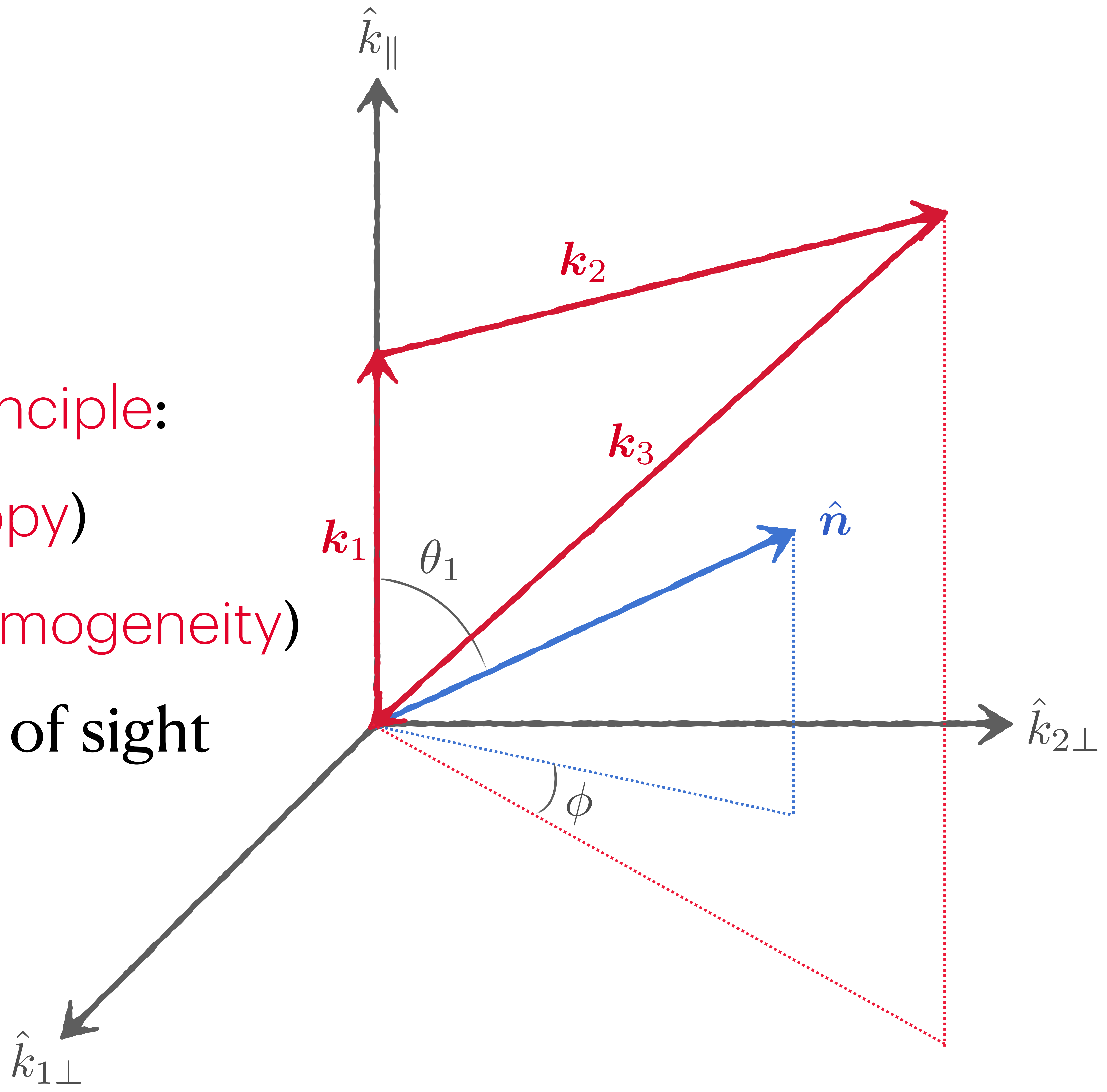
  - −3 translations (homogeneity)
  - +2 angles w.r.t. line of sight
- 5 dof



Comparing bispectra

[SC 2024 (in prep.)]

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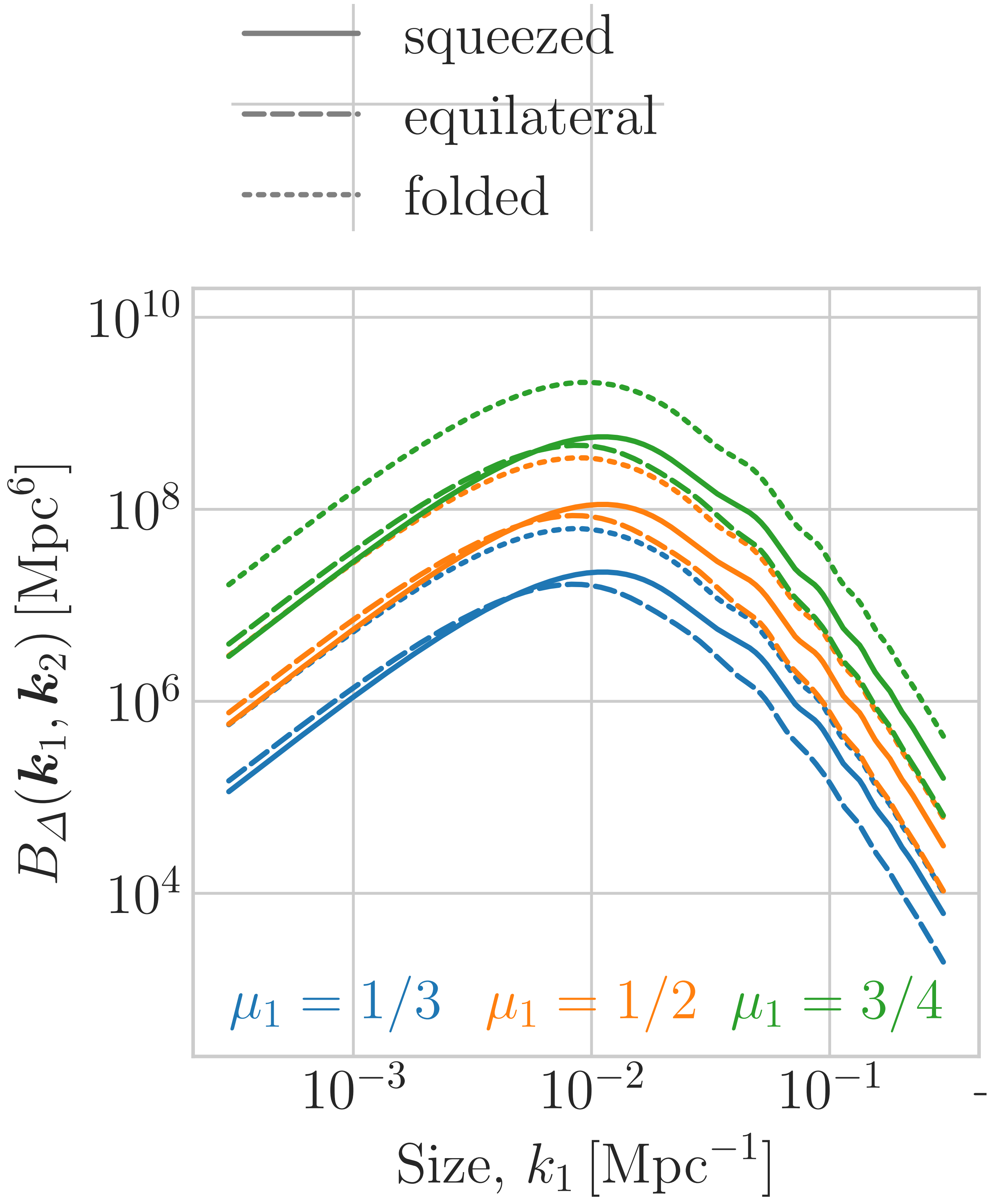
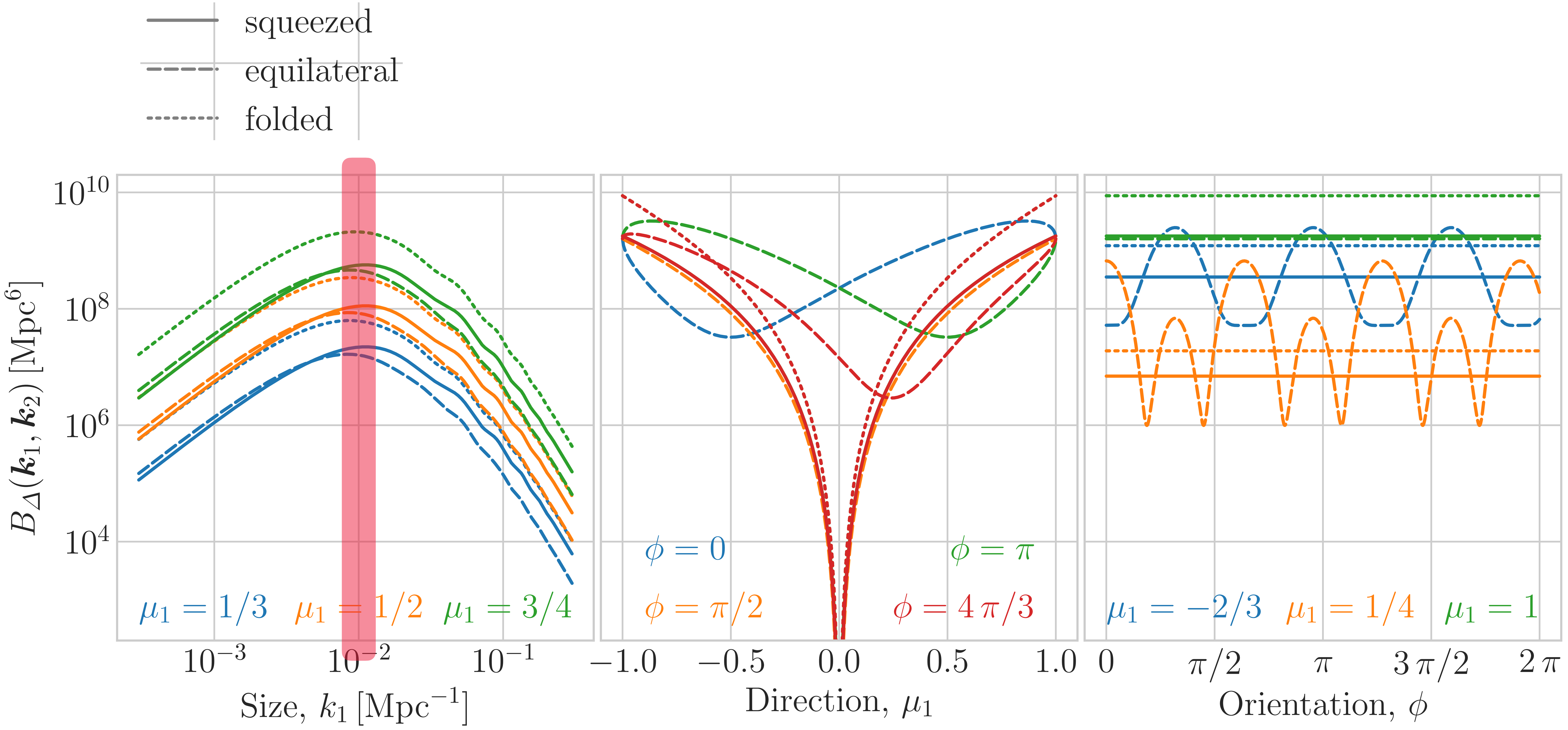


Comparing bispectra

[SC 2024 (in prep.)]



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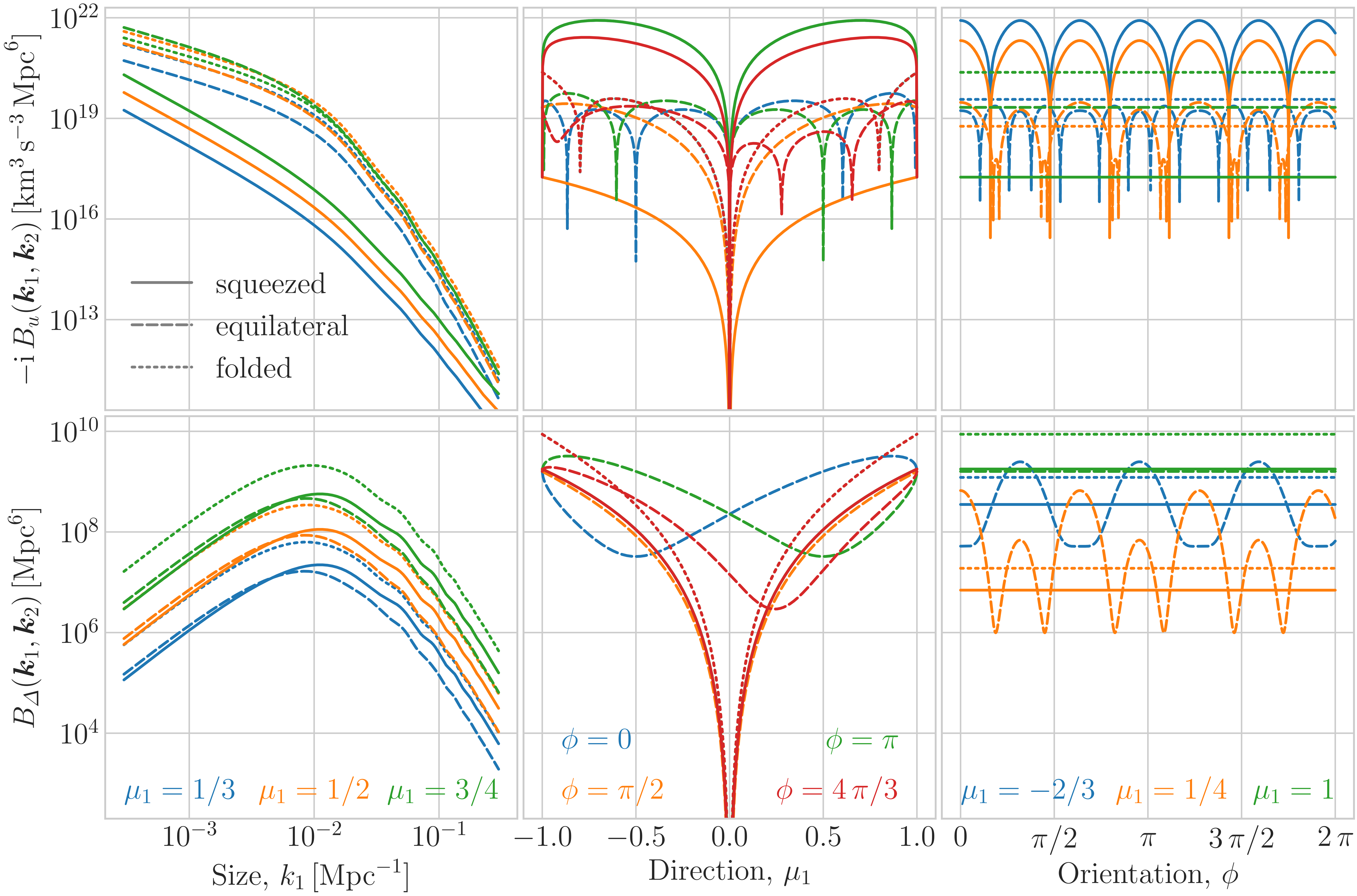

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Detectability

$$\tilde{P}_X = P_X + \begin{cases} 1/\bar{n}_{\rm g}(\bar{z}_i) & \text{if } X = \varDelta \\ \sigma_{\rm v}^2/\bar{n}_{\rm v}(\bar{z}_i) & \text{if } X = \{u, p\} \end{cases}$$

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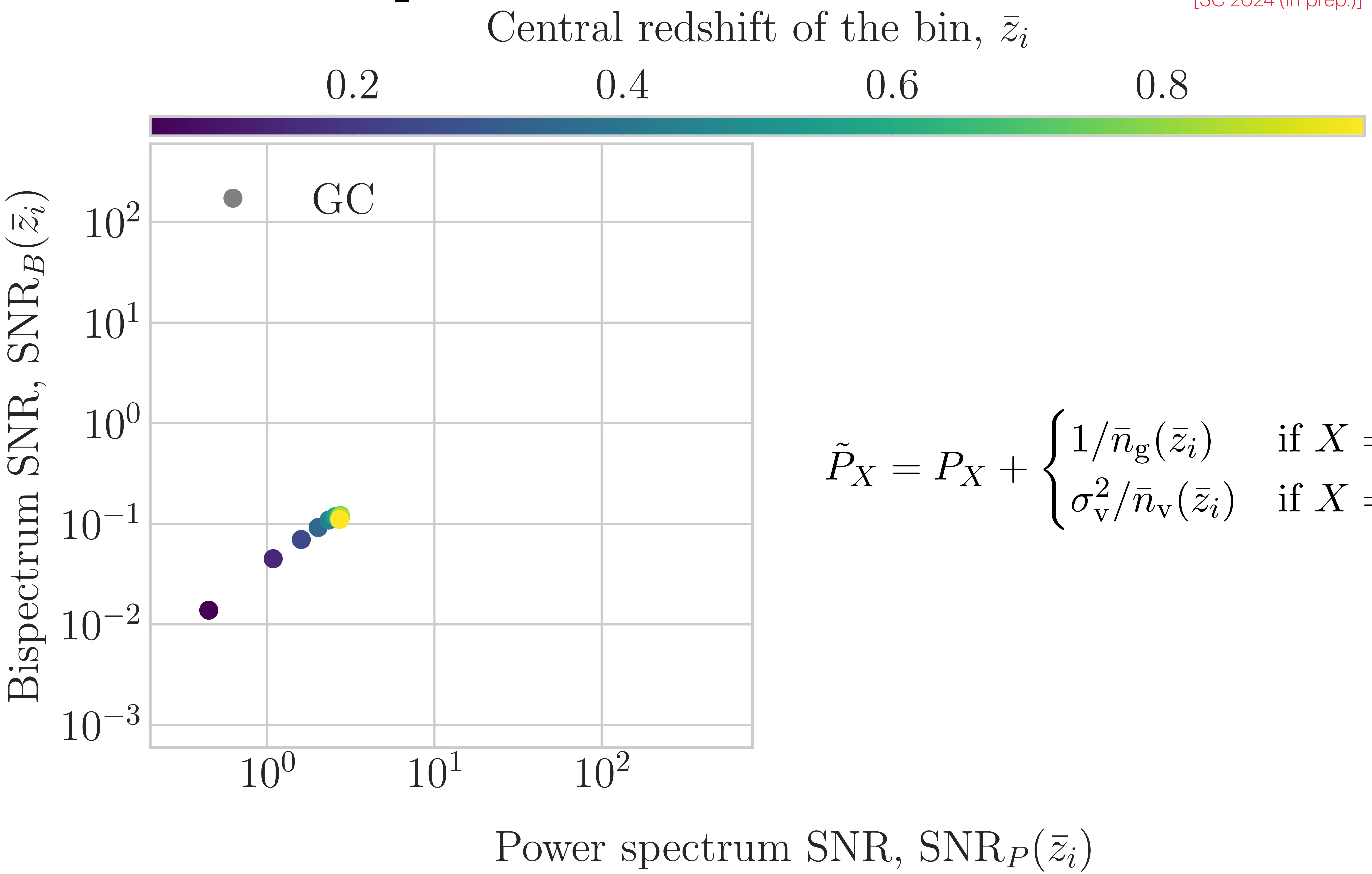

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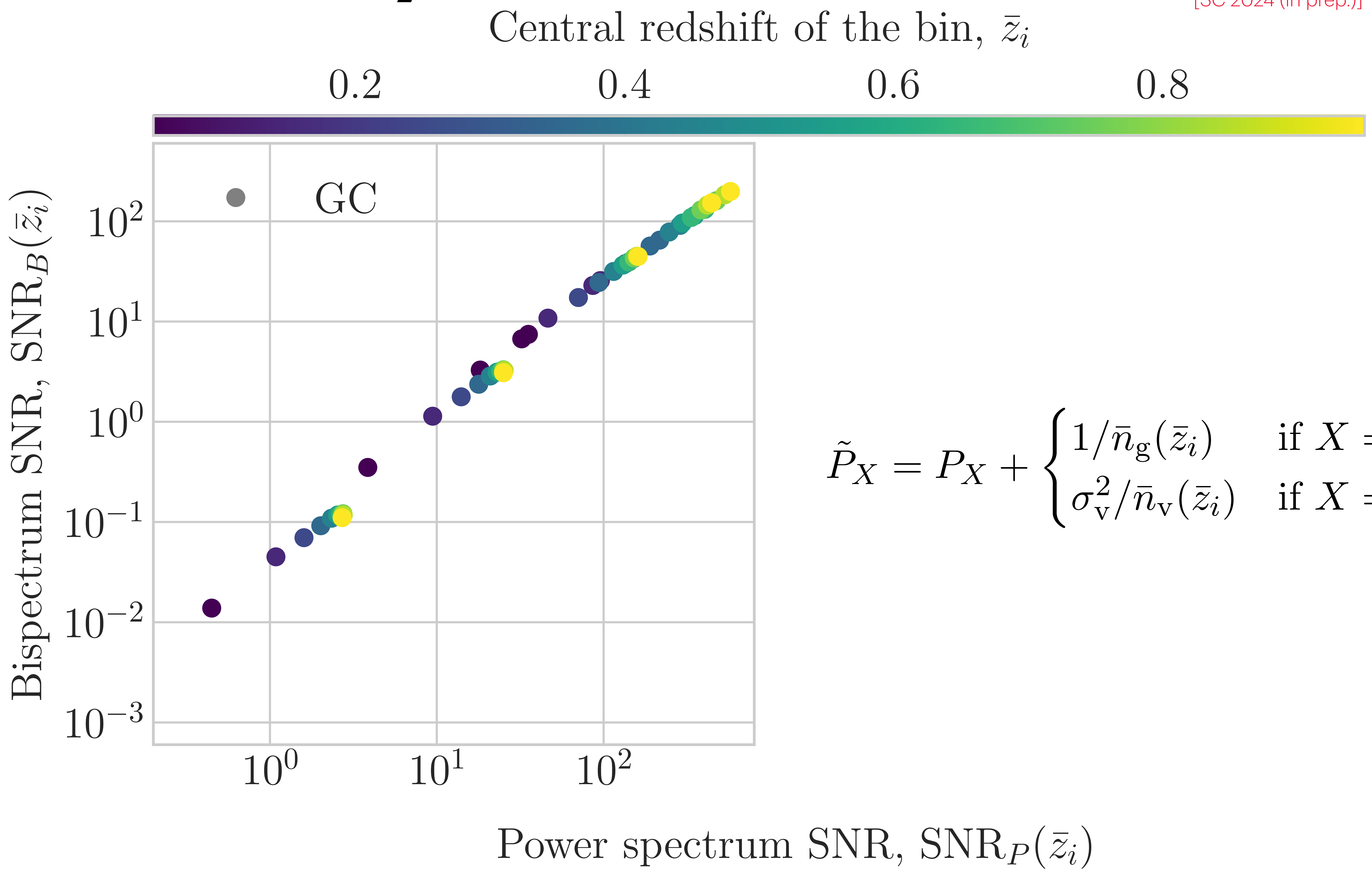

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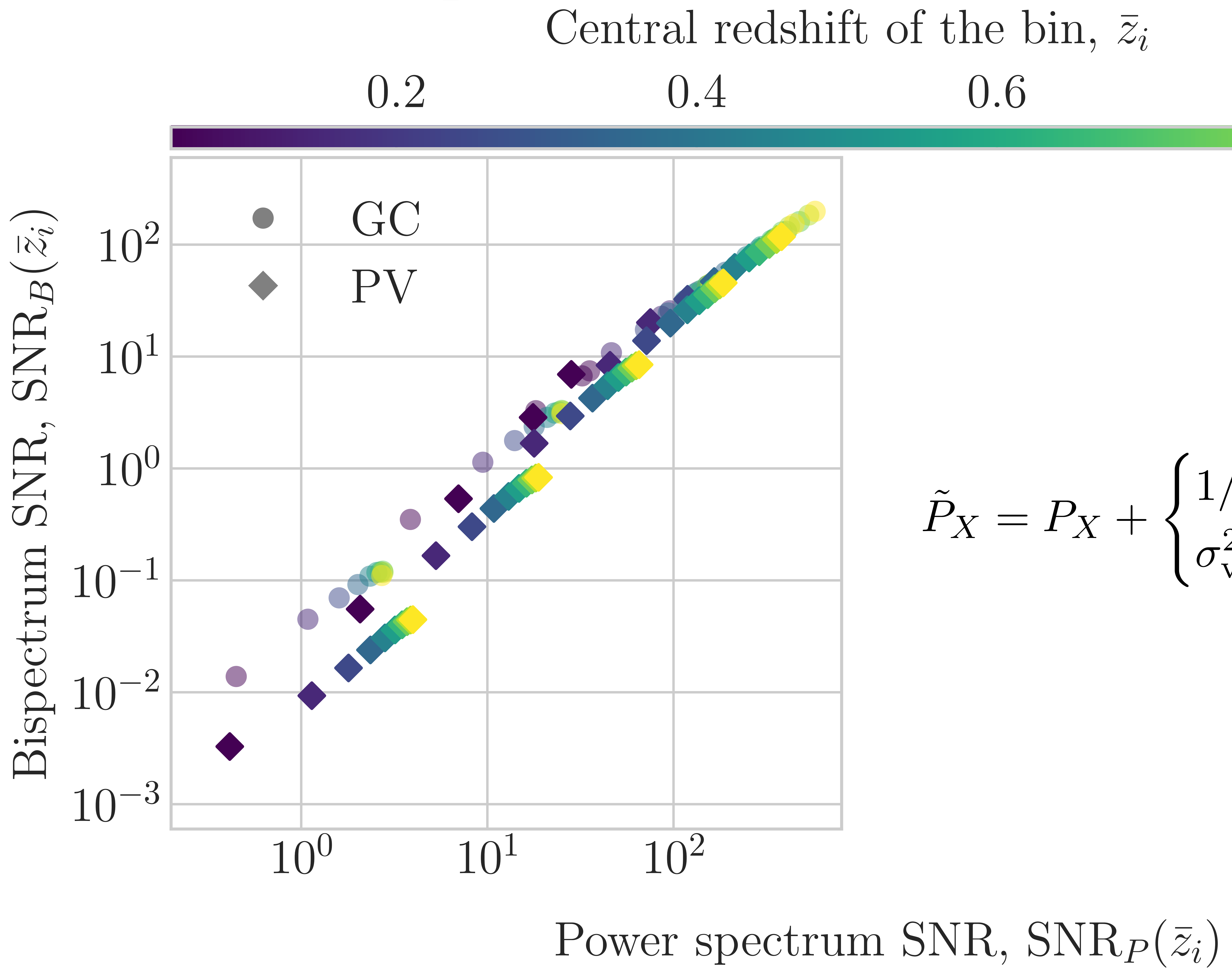

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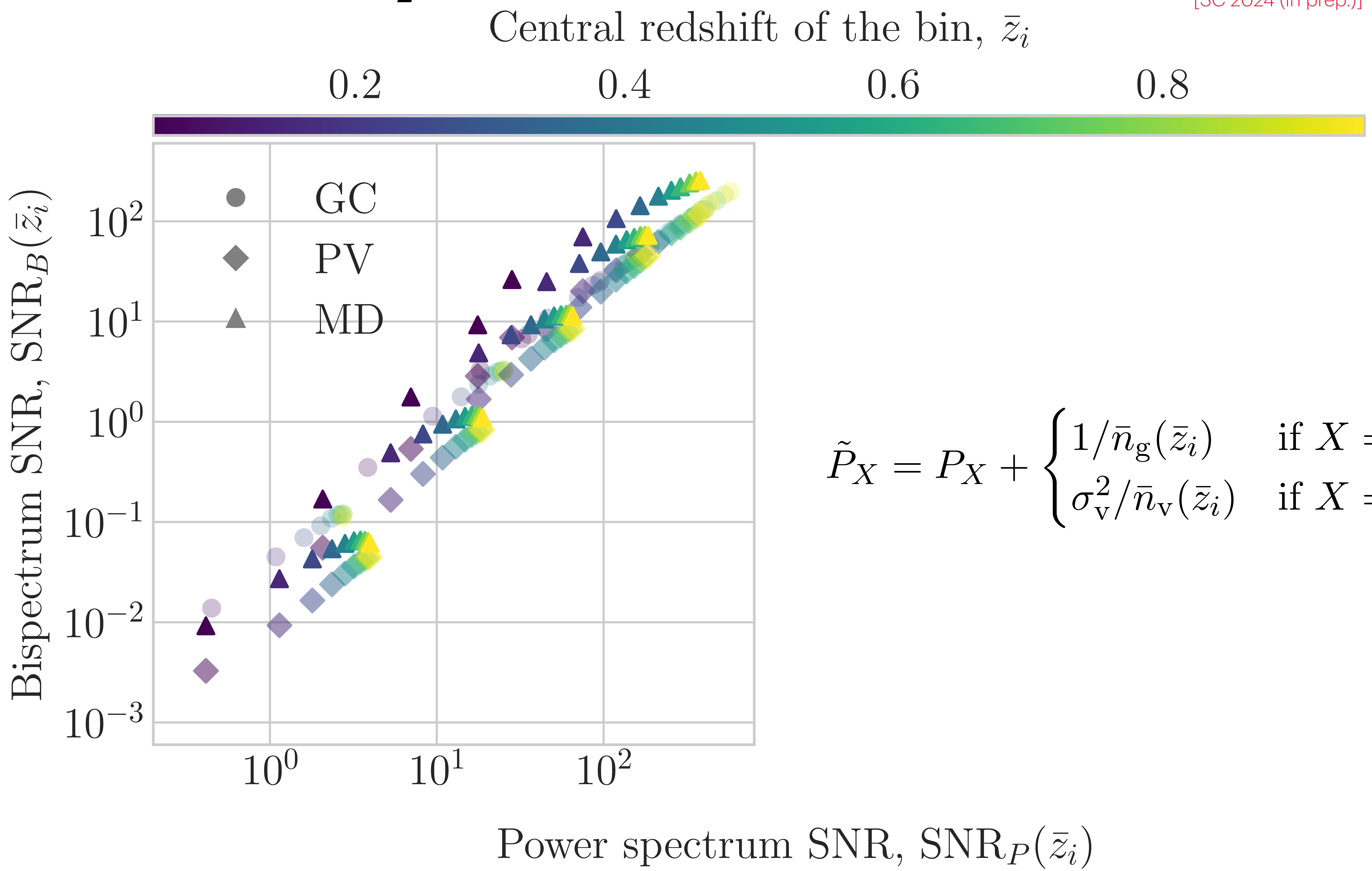

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Information content

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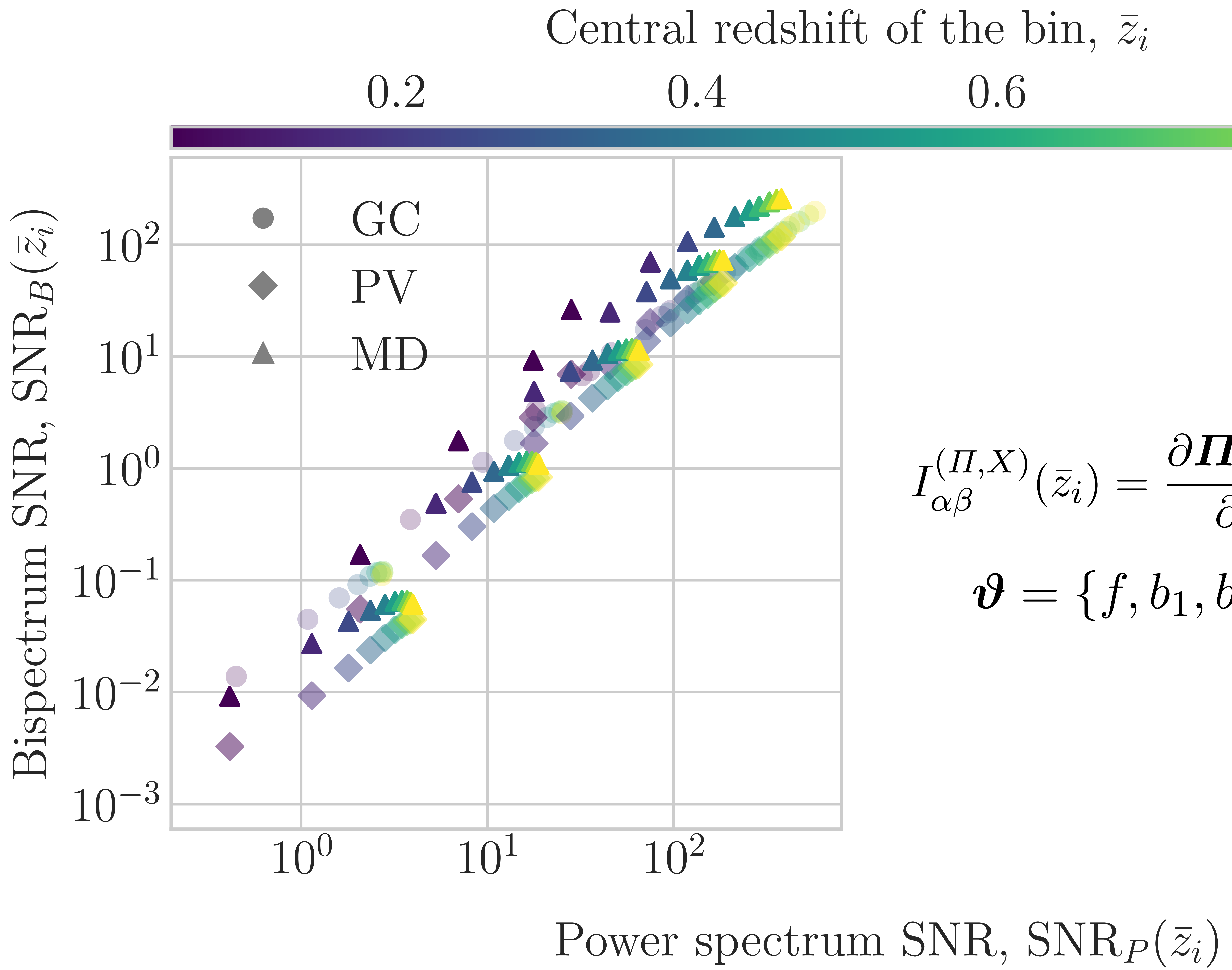

$$I^{(\Pi,X)}_{\alpha\beta}(\bar{z}_i) = \frac{\partial \boldsymbol{\Pi}^{\mathsf{H}}_X(\bar{z}_i)}{\partial \vartheta_\alpha} \mathsf{C}^{-1}(\bar{z}_i) \frac{\partial \boldsymbol{\Pi}_X(\bar{z}_i)}{\partial \vartheta_\beta}$$

$$\boldsymbol{\vartheta} = \{f, b_1, b_2, b_{\mathcal{G}_2}, P_{\rm shot}, B_{\rm shot}\}$$

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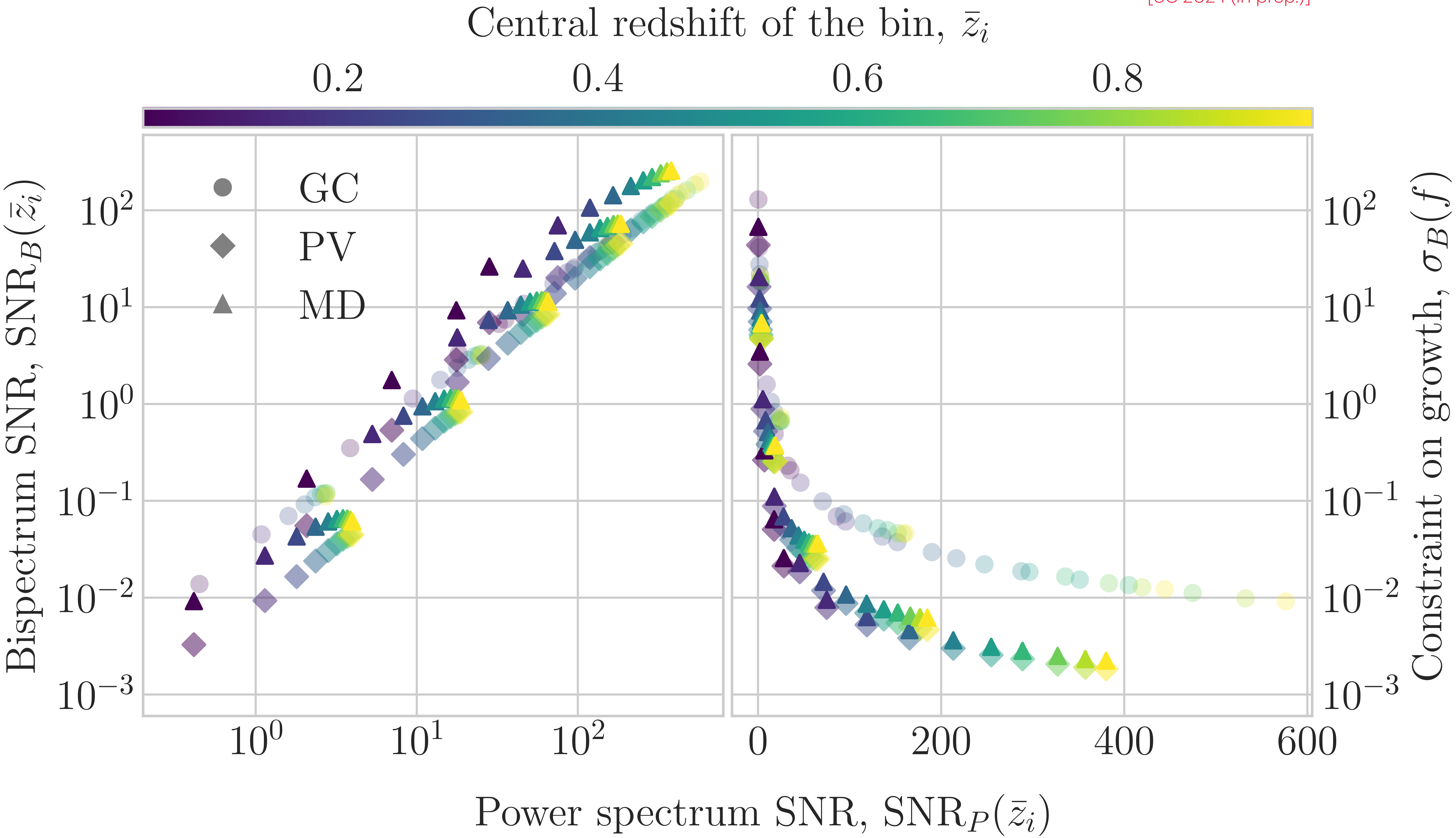

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