

Robustness of different electoral systems to external attack

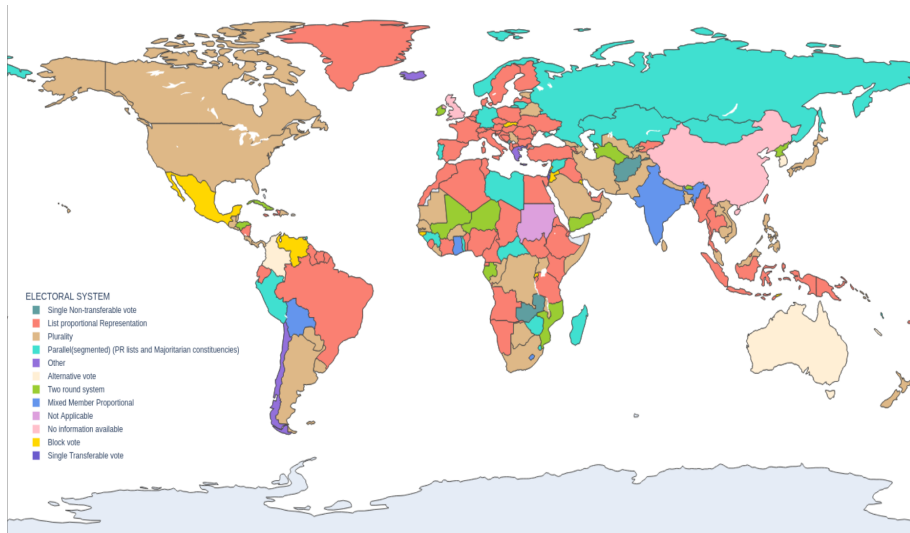
Glory Mary Givi

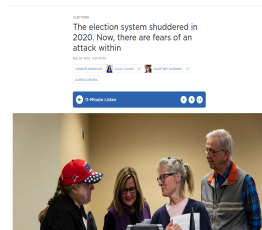
Young Researcher's Day

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Electoral systems





Problem Statement and Construction of the Model

- Are certain democratic systems more robust against such external influence than others?

Ingredients of the model :

- Agents : Representing voters
- Natural opinion : opinion that each agent would have if it were not interacting with other voters.
- Communication distance : opinion difference beyond which voters do not interact with one another.
- An influence field : to model the external attack.

Mathematical model (Two party system)

The natural opinions of agents lies in the interval ($\mathbf{x}_0 \in [-1, 1]^N$).

Opinion of agents evolves according to the following ODE

$$\dot{\mathbf{x}}(t) = -D^{-1}\mathbb{L}_\epsilon \mathbf{x}(t) - (\mathbf{x}(t) - \mathbf{x}_0 - \boldsymbol{\omega}), \quad (1)$$

- D^{-1} : inverse of degree matrix
- $\mathbb{L}_{\epsilon,ij} \neq 0$ if $|x_{0,i} - x_{0,j}| < \epsilon$
- $\boldsymbol{\omega}$: external influence

Mathematical model (Two party system)

With no external influence, the election's outcome is the long term solution of eqn. (1)

$$\mathbf{x}^* = (D^{-1}\mathbb{L}_\epsilon + \mathbb{I})^{-1}\mathbf{x}_0 \quad (2)$$

Outcome:

$$O(\mathbf{x}^*) = \sum_i \text{sign}(x_i^*) = \begin{cases} > 0 & \text{Right party wins,} \\ < 0 & \text{Left party wins,} \end{cases} \quad (3)$$

With influence ($\omega \neq 0$), we have

$$\mathbf{x}^{**} = (D^{-1}\mathbb{L}_\epsilon + \mathbb{I})^{-1}(\mathbf{x}_0 + \omega). \quad (4)$$

We search for the smallest effort $\xi = \|\omega\|_1$ such that $O(\mathbf{x}^*) \neq O(\mathbf{x}^{**})$

Electoral Units and Electoral systems

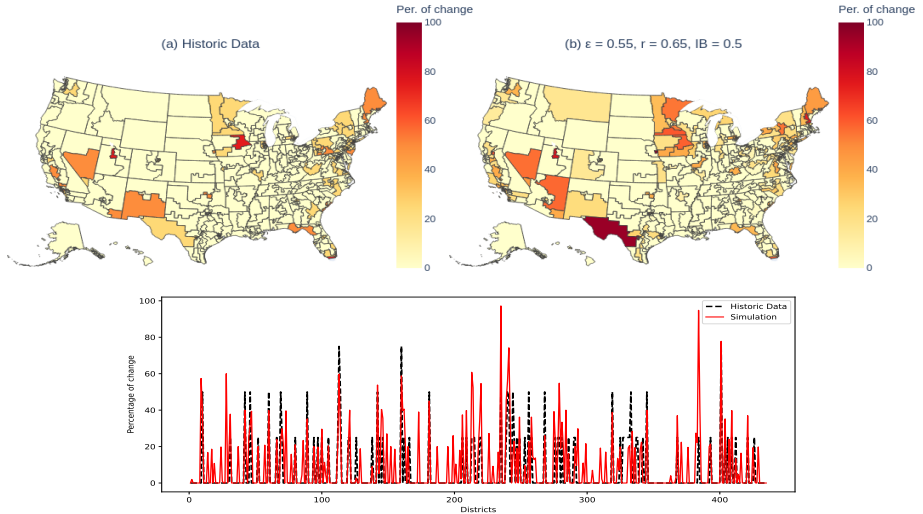
Electoral unit denotes the level in which the votes are counted, this can be Country, States or Districts.

Example :

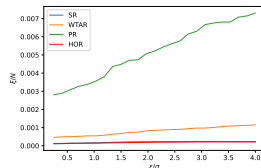
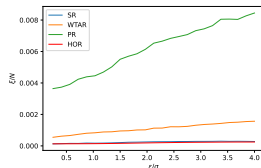
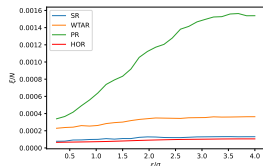
- French presidential election (At country level)
- House of Representative election (At district level)

Suppose there are m seats in an electoral unit. The seats can be distributed proportionally among the parties, or the winner gets all the seats.

Validation : US House of Representative elections



Robustness : Two party system



- Most robust : Proportional Representation (Eg: Swiss National Council elections)
- Robustness increases with increase in communication distance ϵ .
- Model can be extended for higher number of parties.
- Preprint coming out soon....!!



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